

Problems

Normal Load

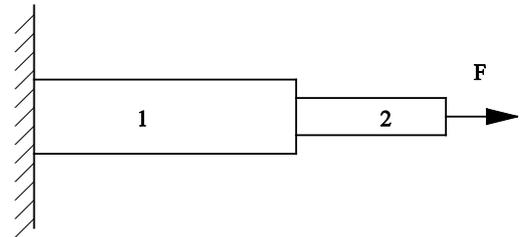
1. A wire of high strength steel (length $l = 20 \text{ cm}$, Young's modulus $E = 210 \text{ GPa}$) is extended by $\Delta l = 0,5 \text{ mm}$ under the influence of a force $F = 10 \text{ kN}$.

- How large is the strain ϵ in the longitudinal direction of the rod?
- Calculate the tensile stress σ in the wire.
- What is cross-sectional area A of the wire?

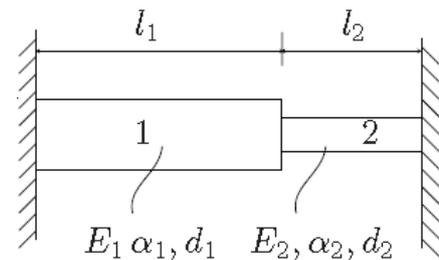
How large is the diameter d of the wire, provided it has a circular cross-section?

2. The shown mechanical system consists of two bars (lengths: $l_1 = 10 \text{ cm}$, $l_2 = 8 \text{ cm}$, diameter: $d_1 = 3 \text{ cm}$, $d_2 = 2 \text{ cm}$, Young's moduli: $E_1 = E_2 = 210 \text{ GPa}$). The force $F = 20 \text{ kN}$ is applied at the right end.

Determine the total elongation.



3. The illustrated mechanical system consists of two rods of circular cross section (diameter d_1 and d_2 , modulus of elasticity E_1 and E_2 , thermal expansion coefficient α_1 and α_2) which are mounted between two rigid plates. The rods were inserted stress free at room temperature. Thereafter, the bars were heated by different temperature increments ΔT_1 and ΔT_2 .



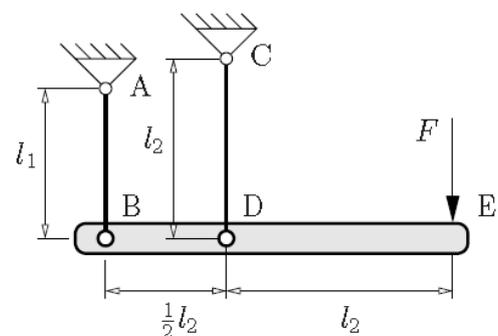
(a) Derive the equations for the stresses in the two bars as a function of ΔT_1 , ΔT_2 , α_1 , α_2 , E_1 , E_2 , l_1 , l_2 , d_1 and d_2 .

(b) Now insert the following numerical values:

$$l_1 = 30 \text{ cm}, l_2 = 50 \text{ cm}, d_1 = 10 \text{ cm}, d_2 = 8 \text{ cm}, E_1 = 206 \text{ GPa}, E_2 = 147 \text{ GPa}$$

$$\alpha_1 = 1,3 \cdot 10^{-5} \text{ 1/K}, \alpha_2 = 0,6 \cdot 10^{-5} \text{ 1/K}, \Delta T_1 = 20 \text{ K}, \Delta T_2 = 40 \text{ K}$$

4. The rigid lever BDE is supported by two rods AB and CD. AB rod is made of aluminum (Young's modulus E_1) and has a cross-sectional area A_1 . CD bar is made of steel (Young's modulus E_2) and has a cross-sectional area A_2 . At point E, the lever is loaded by a concentrated force F .

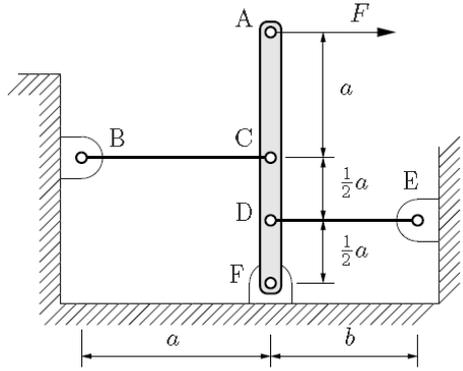


(a) How large are the changes in length of the rods AB and CD?

(b) Determine the lowering of point E at the given load.

Use the values: $F = 30 \text{ kN}$, $l_1 = 300 \text{ mm}$, $l_2 = 400 \text{ mm}$, $E_1 = 70000 \text{ Nmm}^{-2}$, $E_2 = 200000 \text{ Nmm}^{-2}$, $A_1 = 500 \text{ mm}^2$, $A_2 = 600 \text{ mm}^2$

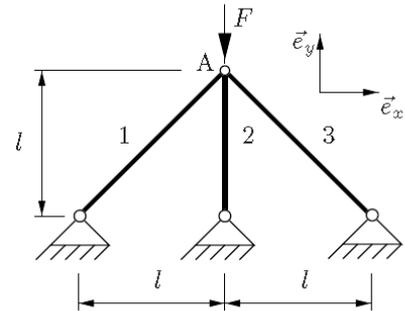
5. The rigid lever AF is supported by two rods BC and DE. Both rods are made of steel (Young's modulus $E = 200 \text{ kNmm}^{-2}$) and have a rectangular cross sectional area ($12 \text{ mm} \times 6 \text{ mm}$). At point A, the lever is loaded by a single force F .



- (a) Is the rigid lever AF mounted statically determined? Can the forces in the Rods BC and DE be determined only from the equilibrium conditions?
- (b) Determine the forces in the two bars?
- (c) Determine the deflection of point A under the specified load.

Given: $F = 2,5 \text{ kN}$, $a = 100 \text{ mm}$, $b = 125 \text{ mm}$, $E = 200 \text{ kNmm}^{-2}$, $A = 72 \text{ mm}^2$

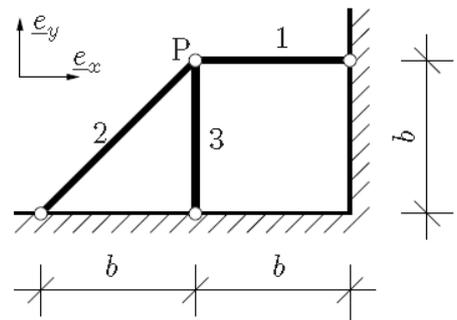
6. The flat, symmetrical tripod consists of three elastic rods. All three bars have the Young's modulus E . The bars 1 and 3 have the cross-sectional area A , the rod 2 has the cross-sectional area $2A$. The tripod is loaded by a force F in the upper hinge point, where all the bars are hinged. Buckling of the rods is excluded. The deformations are very small and purely elastic.



- (a) Calculate the bar forces S_1, S_2 and S_3 .
- (b) What is the displacement \bar{u}_A of the point A ?

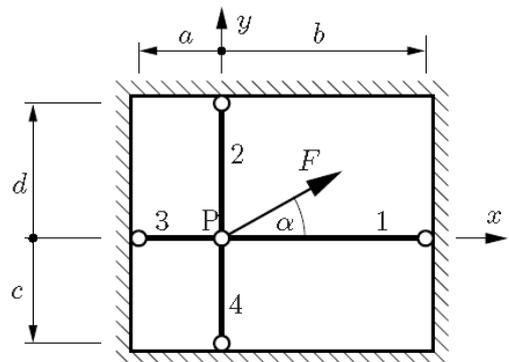
Given: F, A, E, l

7. Rod 1 of the construction shown is heated by ΔT . Calculate the components u_x and u_y of the displacement of the node P. (Note the drawn vector base.)



Given: $b, \Delta T$, Cross sectional area $A = \text{const}$, E , Coefficient of thermal expansion is α_T .

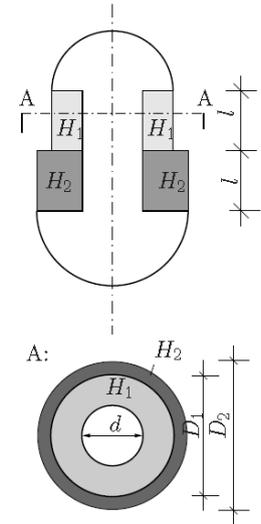
8. The sketched system of four elastic rods is loaded in the central node P with force F in the direction indicated. All components have the Young's modulus E and a square section with edge length D . The lengths are given in the sketch.



Determine the x and y components of the displacement of the point P? The shift should be small and buckling excluded.

Given: $a, b, c, d, D, F, \alpha, E$

9. For connection of the two sleeve rings H1 ($E_1 = 2,1 \cdot 10^5 \text{ N/mm}^2$, $D_1 = 40 \text{ mm}$) and H2 ($E_2 = 0,8 \cdot 10^5 \text{ N/mm}^2$, $D_2 = 50 \text{ mm}$), a rivet ($E_N = 2,1 \cdot 10^5 \text{ N/mm}^2$) is hit through the bore ($d = 20 \text{ mm}$) at a temperature of $T_0 = 520 \text{ K}$.

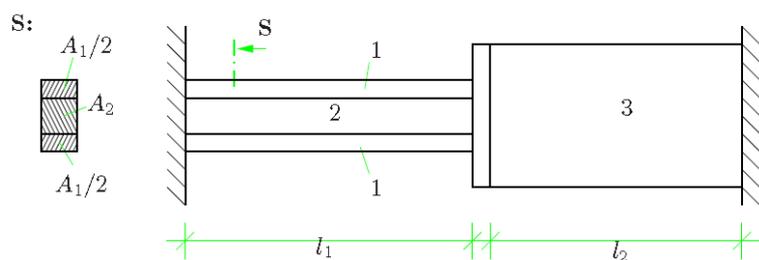


Calculate the stresses in the sleeve rings H1 and H2, as well as in the shaft of the rivet, after cooling to $T_1 = 290 \text{ K}$.

(Assumption: the rivet heads are rigid, the sleeves are cold from the beginning: 290 K).

Use the value: $\alpha_t = 12 \cdot 10^{-6} \text{ 1/K}$

10. The sketched rod is in its right part 3 made of a homogeneous material, in its left part (1 and 2) of a symmetrically constructed composite body.



There is a rigid plate between the parts of the rod. The rod is initially free of stress between two fixed abutments. Then part 3 of the rod is heated by a temperature increment $\Delta\theta$.

- What are the normal stresses in the three cross-section parts?
- What is the displacement of the rigid plate

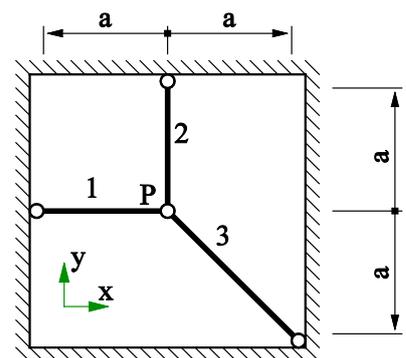
Use the values.: $l_1 = 4,0 \text{ m}$, $l_2 = 3,5 \text{ m}$, $A_1 = 300 \text{ cm}^2$, $E_1 = 2 \cdot 10^4 \text{ N/mm}^2$, $A_2 = 100 \text{ cm}^2$, $E_2 = 2 \cdot 10^5 \text{ N/mm}^2$, $A_3 = 700 \text{ cm}^2$, $E_3 = E_1 = 2 \cdot 10^4 \text{ N/mm}^2$, $\alpha_{t3} = 12 \cdot 10^{-6} \text{ 1/K}$, $\Delta\theta = 40 \text{ K}$

11. Problem (extra)

Three bars are installed as sketched free of stress in a rigid frame made of the same material.

Calculate the stresses in the rods and the displacement of the central node P, caused by a temperature change of ΔT .

Given: a , ΔT , A , E , α_T , $\alpha_T > 0$

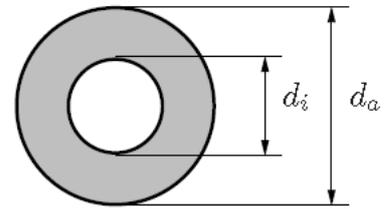


Torsion

12. A preliminary design for connection of a shaft of a motor to a generator provides a hollow shaft with inner diameter $d_i = 100 \text{ mm}$ and outer diameter $d_a = 150 \text{ mm}$.

The maximum allowed shear stress is $\tau_{allow} = 85 \text{ MPa}$. What maximum torque can be transmitted by the shaft, if

- the shaft is made as planned,
- a solid shaft of equal mass is made,
- a hollow shaft of the same mass and outer diameter $d_a = 200 \text{ mm}$ is manufactured?



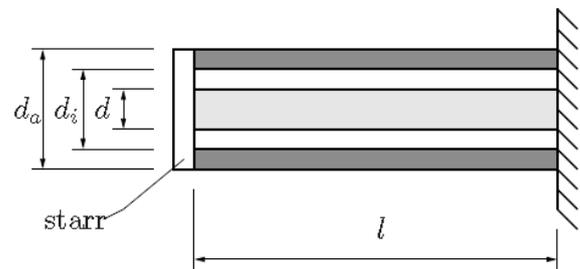
13. A solid steel shaft (diameter d) and a hollow shaft made of aluminum (outer diameter d_a , wall thickness t) are fixedly clamped on the right and are connected by a rigid plate at the left end.

What is the maximum allowed torque that can be applied to the rigid disk, if the allowable shear stress amount for the steel shaft $\tau_s = 120 \text{ MPa}$ and for the aluminum shaft $\tau_A = 70 \text{ MPa}$?

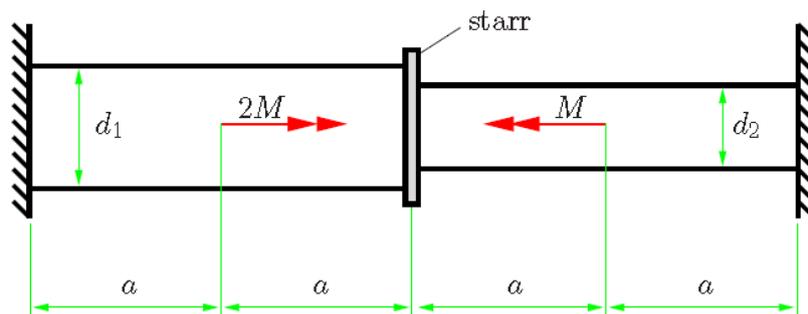
Given:

Solid steel shaft: $d = 50 \text{ mm}$, $G_s = 80 \text{ GPa}$, $\tau_s = 120 \text{ MPa}$,

Hollow shaft made of aluminum: $t = 8 \text{ mm}$, $d_a = 76 \text{ mm}$, $d_i = 60 \text{ mm}$, $G_A = 27 \text{ GPa}$, $\tau_A = 70 \text{ MPa}$



14. Shown is a composite cylindrical rod made of elastic material with shear modulus G which is loaded through two external torques.



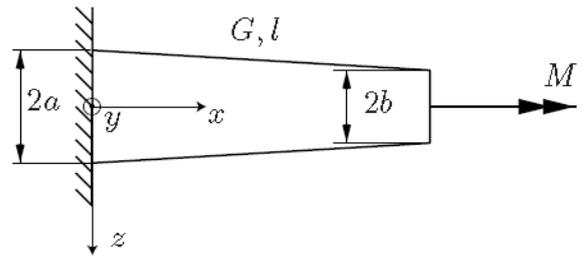
Determine such $\xi = d_1 / d_2$, so that the maximum stresses magnitudes are equal to the left and to the right of the rigid disk.

15. Shown is a rod with a round cross section, in which a is only slightly larger than b .

(a) Determine the polar moment of inertia $I_p(x)$.

(b) Now determine the angle of rotation φ at the right end of the bar!

Given: a, b, M, G, l



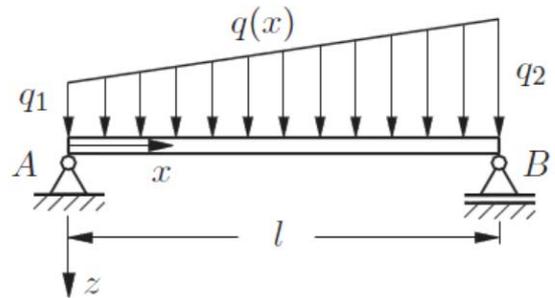
Bending, Deflection of Beams, Bending Line Equation

16. The sketched bar is supported at A and B and is loaded by a linear line load $q(x)$.

(a) Calculate the bending line $w(x)$.

(b) Explain how the maximum deflection \hat{w} can be calculated!

Given: q_1, q_2, l, EI

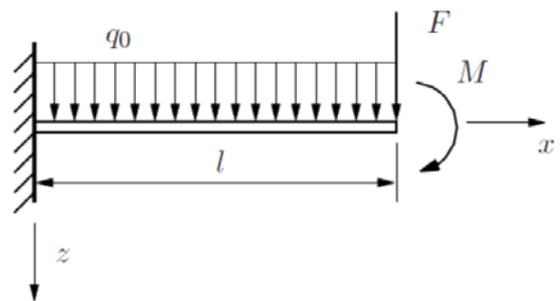


17. The illustrated slender beam (bending stiffness EI , length l) is firmly clamped on the left and is loaded over the entire length by a constant line load q_0 . In addition, at the right end a single force F and a moment M are applied. Show that for deflection \hat{w} and the slope $\hat{\phi}$ of the right end of the beam the following is valid

$$\hat{w} = \frac{Fl^3}{3EI} + \frac{Ml^2}{2EI} + \frac{q_0 l^4}{8EI}$$

$$\hat{\phi} = \frac{Fl^2}{2EI} + \frac{Ml}{EI} + \frac{q_0 l^3}{6EI}$$

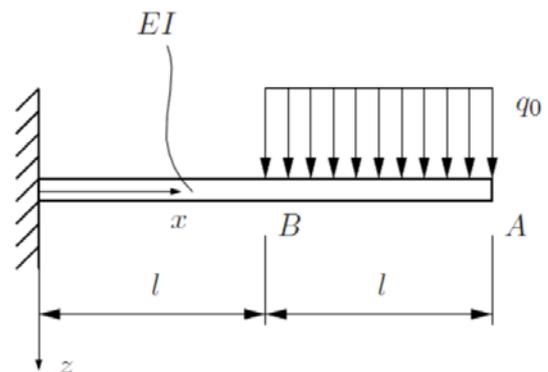
Given: q_0, F, M, l, EI



18. The illustrated slender beam (bending stiffness EI) is firmly clamped on the left and is loaded in the section BA by a constant line load q_0 .

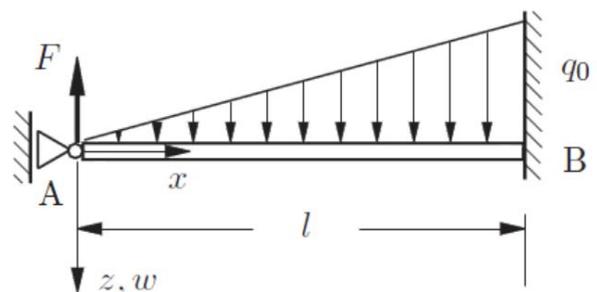
Determine the deflection of point A.

Given: q_0, l, EI



19. The bar shown is fixedly clamped on the right and coupled with a loose bearing to the environment on the left. The beam is loaded by a linear distributed load $q(x)$ and a force F .

(a) What is the differential equation for the deflection $w(x)$?



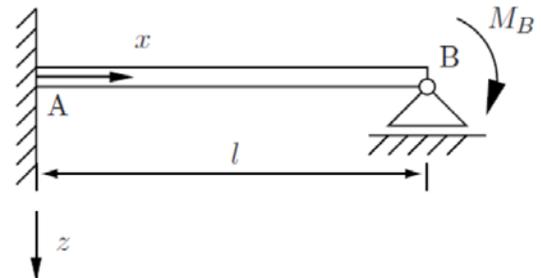
(b) Determine the general solution of the differential equation for the bending line and specify the

geometric and dynamic boundary conditions of the system.

- (c) Determine the unknown constants.
- (d) Determine the angle of rotation φ in the bearing A.
- (e) How the force F has to be chosen so that the deflection $w(0) = 0$?

Given: F, EI, q_0, l

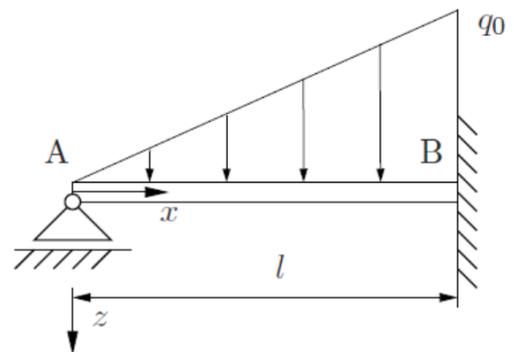
20. The bar shown (bending stiffness EI , length l) is fixedly clamped on the left and coupled with a loose bearing to the environment on the right. The beam is loaded by a moment M_B at B.



- (a) Is the bar statically determinate? Can the internal forces be obtained alone from the equilibrium conditions?
- (b) Determine the support reactions and the the bending moment diagram with the help of the bending line equation.
- (c) Use the principle of superposition to solve the task of part (b).
- (d) What is the maximum bending moment in the beam?

Given: M_s, l, EI .

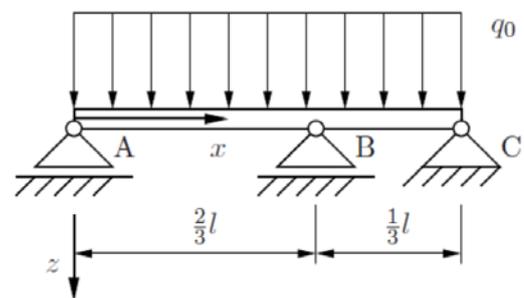
21. The bar shown (bending stiffness EI , length l) is fixedly clamped on the right and coupled with a loose bearing to the environment on the left. The beam is loaded by a linear distributed load $q(x)$.



- (a) Is the bar statically determinate? Can the internal forces be obtained alone from the equilibrium conditions?
- (b) Determine the support reactions in the bearing A.
- (c) What is the deflection $w(x)$ of the beam?
- (d) Determine the angle of rotation φ in the bearing A.

Given: q_0, l, EI

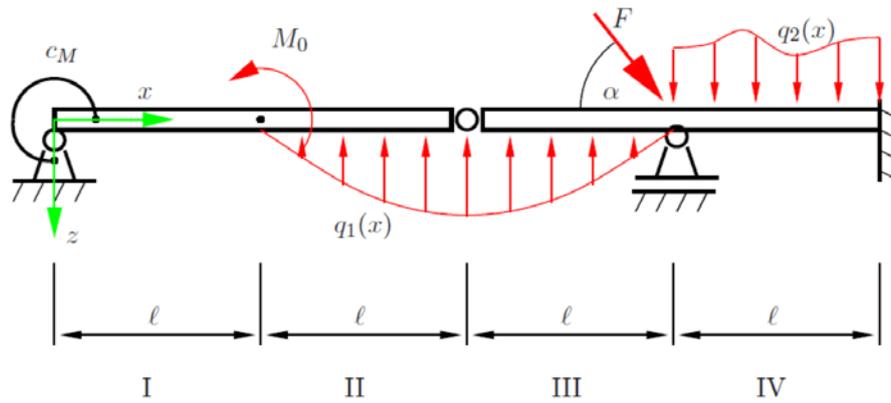
22. The illustrated slender beam (length l , bending stiffness EI) is supported with three bearings. The beam is loaded over the entire length by a constant line load q_0 .



- (a) Is the bar statically determinate? Can the internal forces be obtained alone from the equilibrium conditions?
- (b) Determine the support reactions with the help of the bending line equation.
- (c) Check the reaction force at point B, by calculating it again using the principle of superposition.
- (d) How large is the slope at point A?

Given: q_0, l, EI

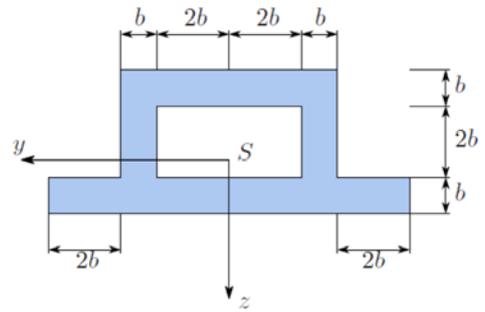
23. Specify all geometric and static boundary and transition conditions of the sketched system!



Moment of Inertia

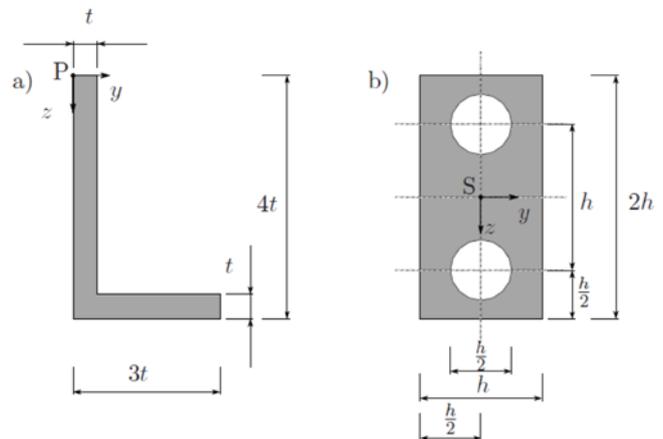
24. The y - and z -axes be central principal axes of the sketched cross section. Determine the axial moment of inertia I_y using the extended table method.

Given: b



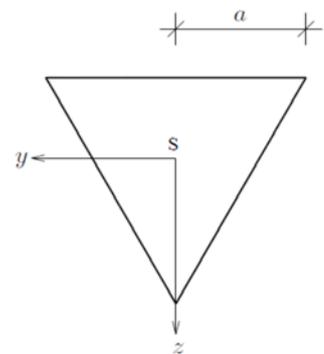
25. Determine the geometrical moments of inertia of the 2nd order (I_{yy} , I_{zz} and I_{yz}) for the cross-sections shown relative to the centroid of the corresponding area.

Given: t, h



26. Using integration, determine the geometrical moment of inertia I_{yy} for the illustrated equilateral triangle.

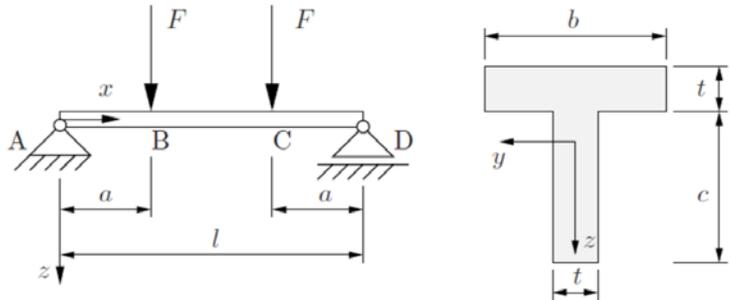
Given: a



Bending, Deflection of Beams, Geometrical Moment of Inertia Stress

27. The illustrated machine part (cross-section as shown) is supported at the end points A and D and is loaded at the points B and C by single forces.

(a) Draw the moment diagram for the beam AD. In which point achieves the bending moment the maximum?



(b) Calculate the centroid of the cross-sectional area.

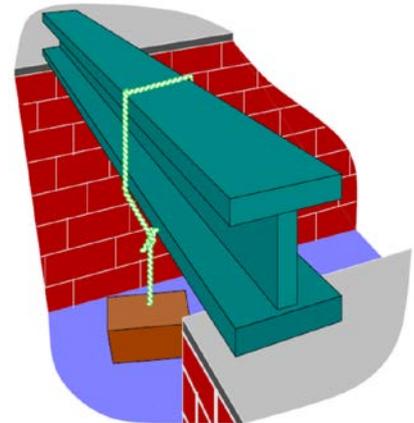
Note: The origin of the coordinate system is drawn in the centroid of the cross-sectional area.

(c) Calculate the geometrical moment of inertia I_{yy} .

(d) How large is the maximum tensile and compressive stress in the machine part?

Given: $F = 15 \text{ kN}$, $a = 250 \text{ mm}$, $l = 875 \text{ mm}$, $t = 12 \text{ mm}$, $b = 100 \text{ mm}$, $c = 75 \text{ mm}$

28. A steel beam rests on two stone walls with the distance l and is loaded in the middle with a force G . The steel beam is welded together from three identical flat steel strips to an I-profile. The flat steel strips have a rectangular cross section with side lengths h and b ($h > b$).



(a) Determine bending moments in the force application point in the middle of the steel beam.

(b) Calculate the maximum normal stress in the cross section

(in the force application point). Use $I = \frac{1}{2}bh^3$ as an

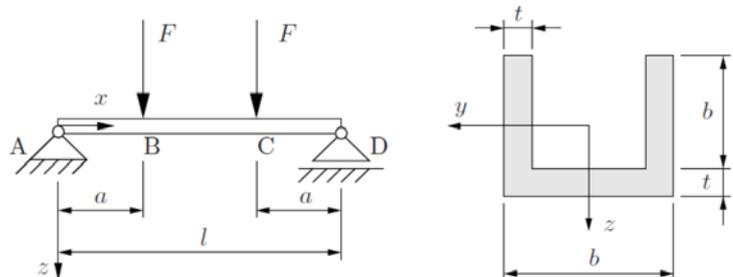
approximation for the geometrical moment of inertia. *Do not use the exact geometrical moment of inertia!*

(c) Prove that the maximum bending moment occurs in the middle of the steel beam.

(d) Show that the real area moment of inertia I_{exact} is larger than the approximation $I = \frac{1}{2}bh^3$.

Given: l , b , h , G

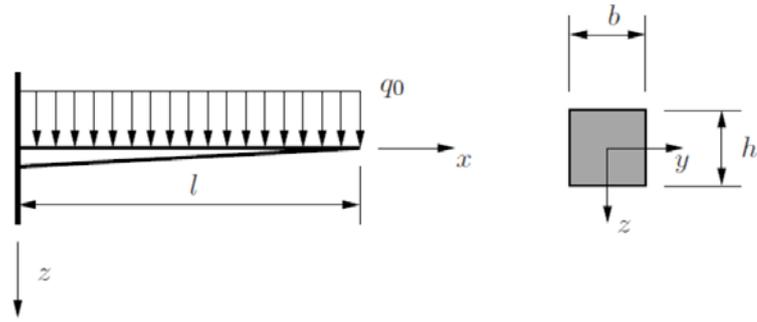
29. The illustrated machine part (cross-section as shown) is supported at the end points A and D and is loaded at the points B and C by single forces. How large is the maximum tensile and compressive stress in the machine part?



Note: The origin of the coordinate system is drawn in the centroid of the cross-sectional area. By calculating the geometrical moment of inertia I_{yy} can be assumed (approximately) $t \ll b$.

Given: F , a , l , t , b

30. The model of a wing spar consists of a one-sided clamped beam of length l . The wing spar is loaded by a constant line load q_0 resulting from the aerodynamic forces and the proper weight. The beam has a rectangular cross-sectional area $A = bh$. The beam height h is a function of the longitudinal coordinate x . The material is isotropic.



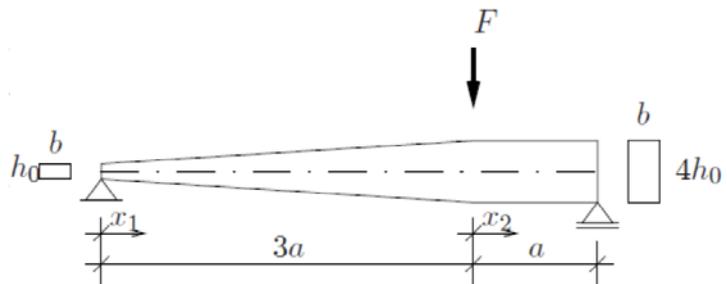
(a) How should the beam height h depend on the coordinate x , so that the maximum value of longitudinal stress σ_{\max} is constant and equal to the allowed stress σ_{allowed} over the whole length of the beam.

(b) Calculate for this case the bending line $w(x)$.

Given: $q_0, l, E, b, \sigma_{\text{allowed}}$

31. Determine the maximum stress $\sigma(x)$ in each cross section x and display it graphically! How large is the maximum normal stress?

Given: b, h_0, a, F



Stresses, Mohr's circle,

32. Given is the plane state of stress $\sigma_{xx} = 7 \frac{N}{mm^2}$, $\sigma_{yy} = 1 \frac{N}{mm^2}$ and $\tau_{xy} = 4 \frac{N}{mm^2}$ in the x, y -coordinate system.

(a) Determine the principal stresses σ_1 and σ_2 .

(b) What is the maximum shear stress τ_{max} ?

(c) Sketch the Mohr's circle of stress for this stress state and identify σ_{xx} , σ_{yy} , τ_{xy} , σ_1 and σ_2 .

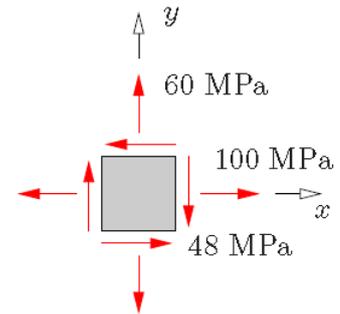
(d) Determine the normal stresses $\sigma_{\xi\xi}$ and $\sigma_{\eta\eta}$ and the shear stress $\tau_{\xi\eta}$ in the ξ, η - coordinate system, which was rotated relatively to the x, y -system by $\varphi = 45^\circ$ around the z -axis and draw these into the Mohr's circle.

33. The stress state at a point in a thin steel plate is shown alongside. Determine

(a) the principal directions and the principal stresses,

(b) the maximum shear stress and

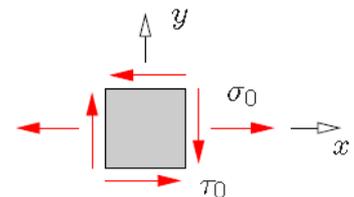
(c) stress components for an element obtained from the illustrated one by rotation by 30° counter clockwise.



34. The studied plane state of stress consists of a Normal stress $\sigma_0 = 100 \text{ MPa}$ in the direction of x -axis and an unknown shear stress τ_0 .

(a) Determine the magnitude of the shear stress τ_0 , so that the largest normal stress is 100 MPa .

(b) What is now the maximum shear stress?

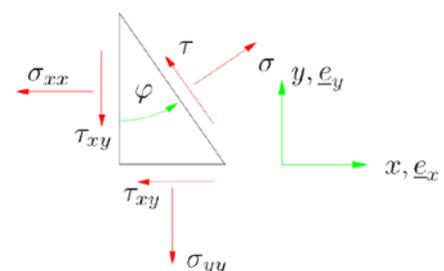


35. The shown plate section is under the influence of the indicated stresses. Deduce in this general case, the equations for the Mohr's circle!

(a) Assume the equilibrium of forces in the x and y -direction and determine equations for σ and τ in a possibly simple form using trigonometric theorems!

(b) Create by squaring and adding the equations an equation of a circle!

(c) Identify the center, radius, the maximum shear stress and the principal stresses!



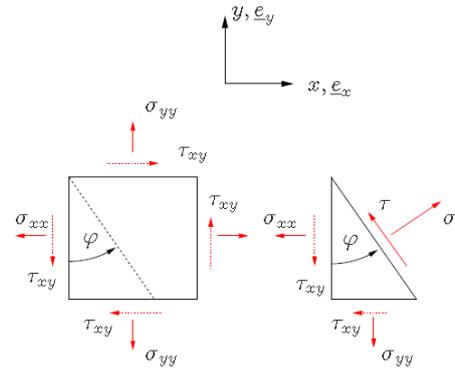
36. The shown rectangular plate is in a plane (so-called. Biaxial) stress condition. It is loaded with the normal stresses $\sigma_{xx} = -5 \text{ N/m}^2$ and $\sigma_{yy} = 21 \text{ N/m}^2$.

(a) Construct the Mohr's circle for this stress state!
 Determine graphically and mathematically normal and shear stress for $\varphi = 60^\circ$!

(b) How large is the maximum shear stress τ_{\max} and the corresponding angle?

(c) How large are the principal stresses σ_{\min} and σ_{\max} and the corresponding angle?

(d) Repeat (a) through (c) for the case that the disc is additionally loaded by a shear stress $\tau_{xy} = 10 \text{ N/m}^2$?

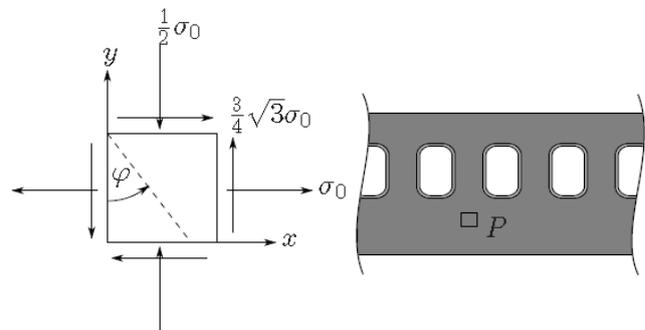


37. The plane state of stress at point P of the surface of an aircraft fuselage is unambiguously characterized by the shown surface element.

(a) Determine the maximum shear stress τ_{\max} in the point P.

(b) How large are the principal stresses and at which angles φ_i^* occur these?

(c) Draw the Mohr's circle of stress (do not forget axis labels) and mark the above calculated characteristic values.



Given: $\sigma_0, \frac{3}{4}\sqrt{3} \approx 1,3$

38. Given is a thin-walled cylinder (radius R , length l , wall thickness t). Due to internal pressure p , the circumferential and longitudinal stresses in the shell are created.

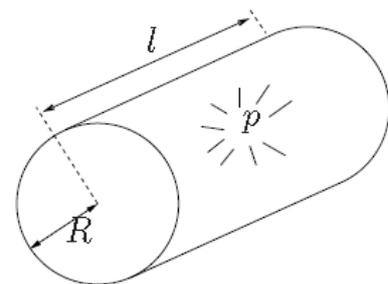
(a) Determine the longitudinal stress σ_{xx} and the circumferential stresses σ_{yy} by a cut and use of equilibrium conditions!

(b) Construct the Mohr's circle!

(c) Determine graphically and analytically the normal and shear stresses for $\varphi = 60^\circ$!

(d) How large is the maximum shear stress τ_{\max} and the corresponding angle?

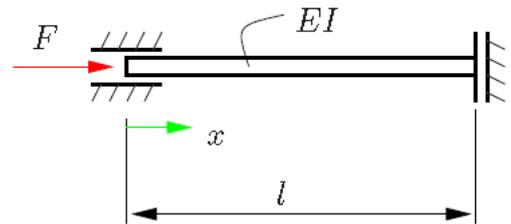
Given: $t = 10^{-3} \text{ m}, p = 0,2 \cdot 10^6 \text{ Pa}, R = 0,5 \text{ m}$



Buckling

39. The buckling condition and the critical load shall be determined for an elastic rod loaded by a compressive force.

Given: l, EI, F

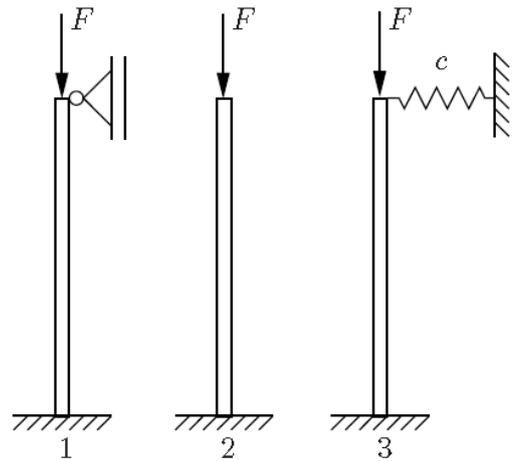


40. A bar with the bending stiffness EI and the length l is mounted in various ways.

(a) Calculate the critical loads F_{crit} for the cases 1-3!

(b) How would the ratio of the beam lengths l_1/l_2 be chosen, so that for the cases 1 and 2 the same critical load is obtained?

Given: F, c, l



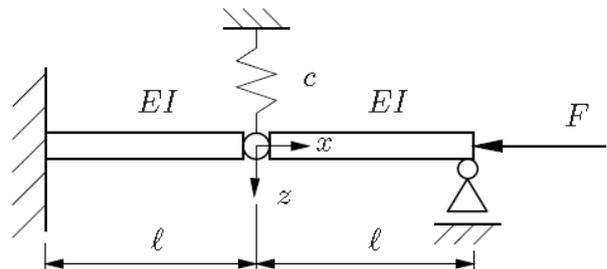
41. Given the system which is mounted and loaded as sketched.

(a) Calculate the critical force for the limiting cases of an infinitely soft and an infinitely stiff spring.

(b) Write the boundary and transition conditions for the system.

(c) Set up the characteristic equation for the calculation of the critical load. (The determinant doesn't need be calculated.)

Given: l, EI, c, F



42. The sketched bar is installed stress free at the temperature T_1 . Now, the rod is uniformly heated to the temperature T_2 .

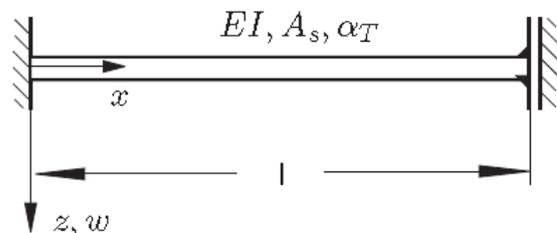
(a) What is the differential equation for the buckling problem (Buckling equation)?

(b) Calculate the force F resulting from the temperature increase.

(c) Determine the critical buckling load F_{crit} .

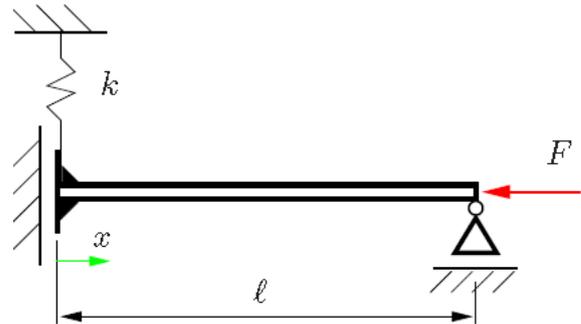
(d) Determine the temperature T_2 at which the rod loses its stability.

Given: EI, A_s, α_T, T_1



43. The beam (length l) shown is loaded with a normal force $F > 0$. The buckling problem should be investigated. In the initial position shown, the spring is relaxed.

- (a) Determine all the necessary boundary conditions.
- (b) Set up the system of equations for calculating the integration constants and determine the eigenvalue equation.



- (c) What is the critical load F_{crit} for the case that the spring is infinitely soft?

Given: l , EI , F , k