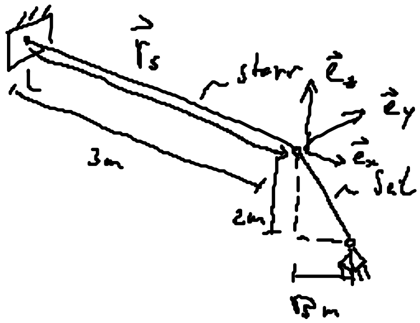
Kraftvektoren

Basisvektoren

$$\vec{F} = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = F_x \, \vec{e}_x + F_y \, \vec{e}_y + F_z \, \vec{e}_z$$

Komponenten



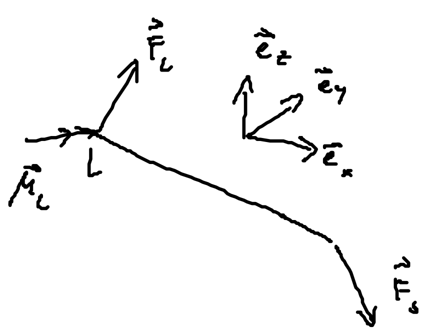
geg: Kraft im Seil sei $F_S = 30\,N$

ges: Lagerreaktionen

1) skalare Lösung: ⇝ Vorstellungskraft

2) vektorielle Lösung: ⇝ Mathe

1) FS:



2) GGB:

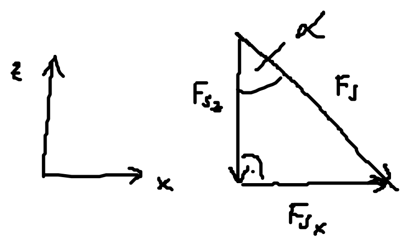
$$\sum \vec{F}_i = 0 \Rightarrow \vec{F}_L = -\vec{F}_S$$

$$\sum \vec{M}_i^L = 0 \Rightarrow \vec{M}_L = -\vec{M}_S^L$$

3.) $\vec{F}_S = ?$

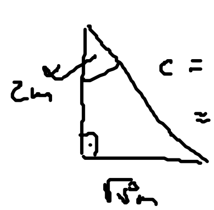
i) $\vec{F}_S = \begin{pmatrix} F_{Sx} \\ F_{Sy} \\ F_{Sz} \end{pmatrix}$

Skizze



$F_{Sx} = F_S \sin(\alpha)$

$F_{Sz} = -F_S \cos(\alpha)$



$c = \sqrt{2^2 + 5}\ m$

$= 3\ m$

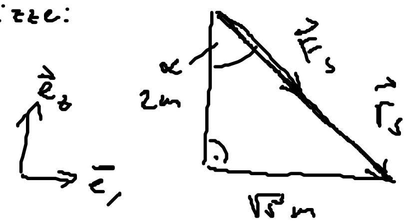
$\sin(\alpha) = \frac{\sqrt{5}}{3}$

$\cos(\alpha) = \frac{2}{3}$

$$\Rightarrow \quad \vec{F}_S = F_S \begin{pmatrix} \frac{\sqrt{5}}{3} \\ 0 \\ -\frac{2}{3} \end{pmatrix}$$

ii) $$\vec{F}_S = \underbrace{F_S}_{\text{Betrag}} \cdot \overbrace{\vec{e}_S}^{\text{Richtung } |\vec{e}_S| = 1}$$

Skizze:



$$\vec{r}_S = \begin{pmatrix} \sqrt{5}\,m \\ 0 \\ -2m \end{pmatrix}$$

$$\vec{e}_S = \frac{\vec{r}_S}{\| \vec{r}_S \|}$$

$$\vec{e}_S = \frac{1}{3\cancel{m}} \begin{pmatrix} \sqrt{5}\,\cancel{m} \\ 0 \\ -2\cancel{m} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{5}}{3} \\ 0 \\ -\frac{2}{3} \end{pmatrix} \Rightarrow \vec{F}_S = F_S \cdot \begin{pmatrix} \frac{\sqrt{5}}{3} \\ 0 \\ -\frac{2}{3} \end{pmatrix}$$

4.) LR:

$$\vec{F}_L = -\vec{F}_S = F_S \begin{pmatrix} -\frac{\sqrt{5}}{3} \\ 0 \\ \frac{2}{3} \end{pmatrix}$$

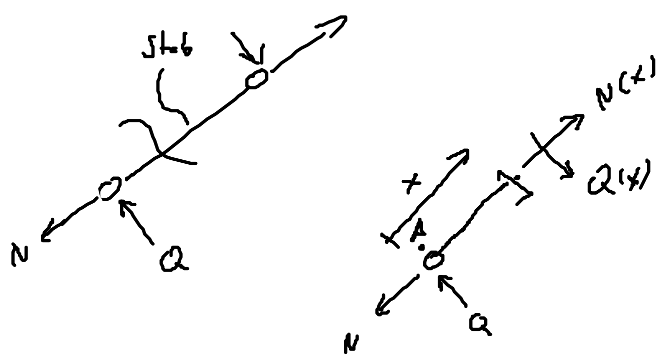
$$\vec{M}_L = -\vec{M}_S^L = -(\vec{r}_S \times \vec{F}_S) = -\Big(3m\, \overbrace{\underbrace{\vec{e}_x \times F_S \Big(\frac{\sqrt{5}}{3} \vec{e}_x}_{=0} + \Big(-\frac{2}{3}\Big) \vec{e}_z}^{= -\vec{e}_y} \Big)\Big)$$

$$\vec{M}_L = -\cancel{3}m\, \frac{2}{\cancel{3}} F_S \vec{e}_y = -2 \cdot F_S\, m\, \vec{e}_y = -60\, Nm\, \vec{e}_y$$

Fachwerke:

ideales FW:

- gelenkig gelagert
- Kräfte nur in Knoten

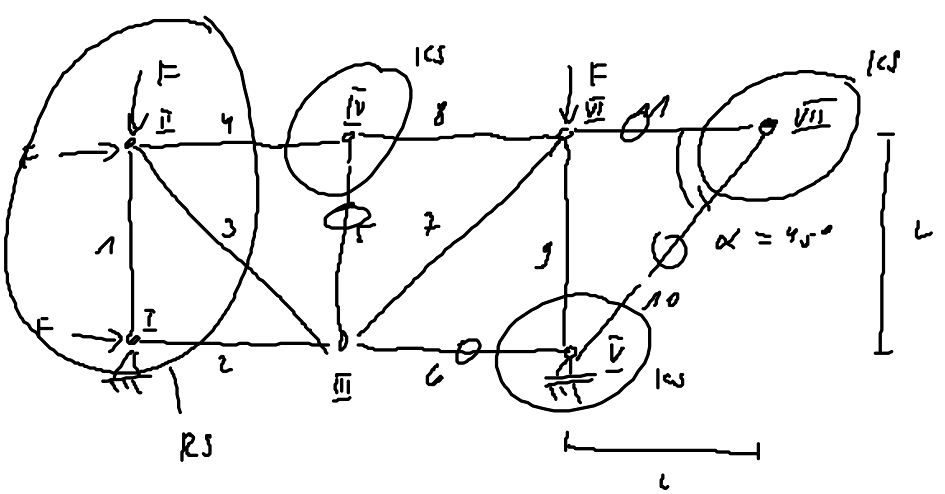


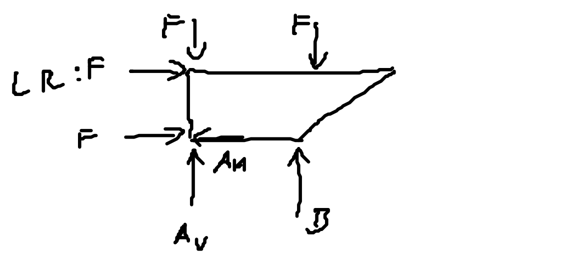
x: $N(x) = N = \text{const.}$

Stäbe übertragen nur $\Leftarrow$ z: $Q(x) = Q = 0$

Normalkräfte

$\curvearrowleft A: \quad Q(x)\cdot x = 0 \quad \Rightarrow \quad Q(x) = 0$

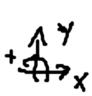


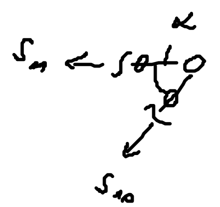


GGB: $B = \frac{3}{2}F, \quad A_V = \frac{1}{2}F, \quad A_H = 2F$

1) Nullstäbe

KS: VII



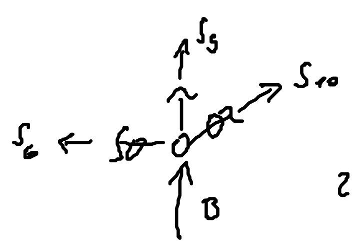


$x: \quad 0 = -S_{11} - S_{10}\cos(\alpha) \Rightarrow S_{11} = -S_{10}\cos(\alpha) = 0$

$y: \quad 0 = -S_{10}\sin(\alpha) \Rightarrow S_{10} = 0$

1. NST-Regel

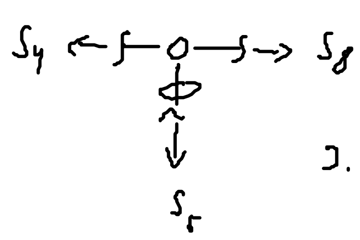
KS: V



$x: \quad 0 = -S_6 \Rightarrow S_6 = 0$

2. NST-Regel

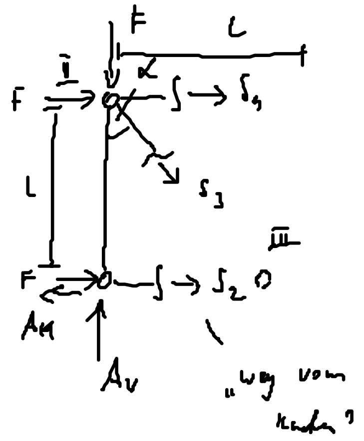
KS: IV



$y: \quad 0 = -S_5 \Rightarrow S_5 = 0$

3. NST-Regel

2) RITTER-Schnitt



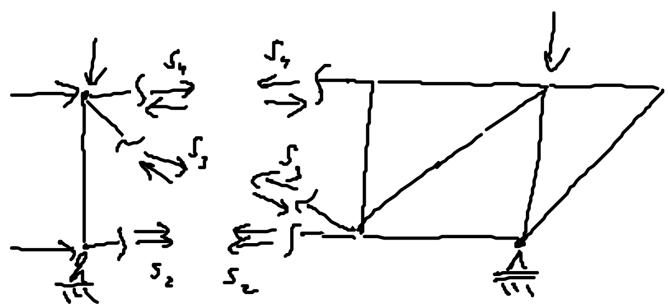
$$M^{II}: \; 0 = -A_V \cancel{l} - \cancel{Fl} - S_4 \cdot \cancel{l} + \cancel{Fl}$$

$$S_4 = -A_V = -\frac{1}{2}F \quad \text{(D)}$$

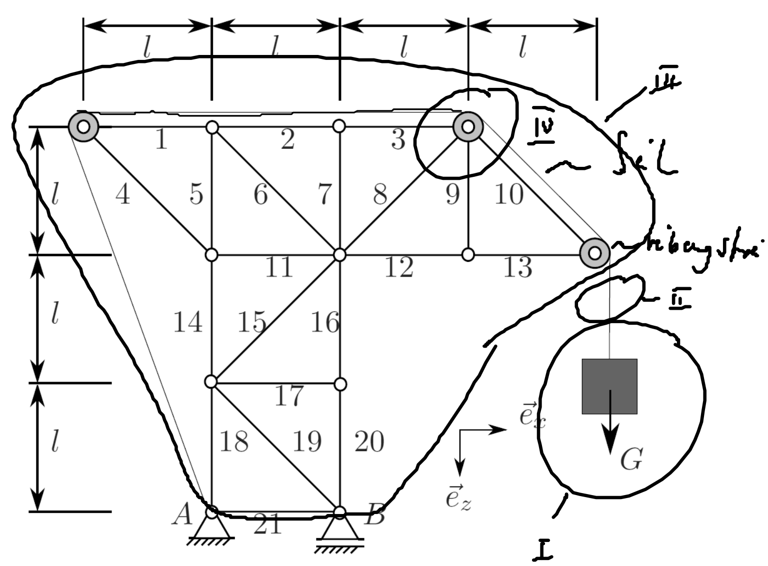
$$y: \; 0 = A_V - F - S_3 \frac{\sqrt{2}}{2}$$

$$S_3 = \frac{2}{\sqrt{2}}\left(A_V - F\right) = -\frac{\cancel{2}}{\sqrt{2}}\frac{1}{\cancel{2}}F = -\frac{1}{\sqrt{2}}F \quad \text{(D)}$$

$$M^{I}: \; 0 = S_2 \cdot \cancel{l} + F\cancel{l} - A_H \cdot \cancel{l} \Rightarrow S_2 = A_H - F = +F \quad \text{(Z)}$$

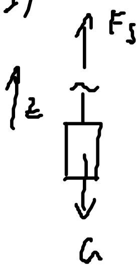


Seile im System



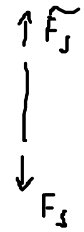
Seil:
- Nur Kräfte entlang seiner Richtung
- Nur Zugkräfte
- biegeweich

I)



$z: F_S = G$

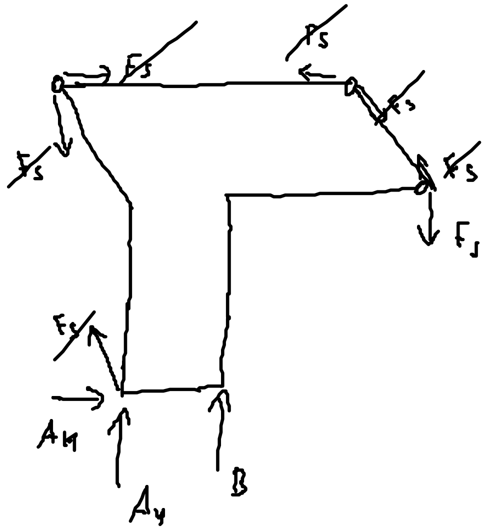
II)



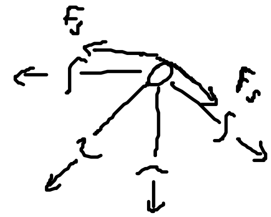
$z: \tilde{F}_S = F_S = \text{const.}$

Seilkraft im ganzen Seil: F_S

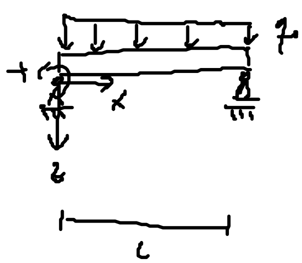
III)



IV)



Schnittlasten



ges: Q, M

$\Rightarrow$ SL-Differentialgleichungen

$$\frac{dQ}{dx} = Q' = -q \qquad (1)$$

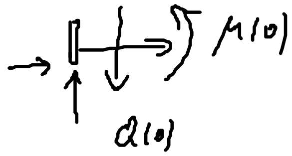
$$\frac{dM}{dx} = M' = Q \qquad (2)$$

(1): $Q(x) = -\int q(x)\,dx + C_1 = -q_0 x + \underline{C_1}$

(2): $M(x) = \int Q(x)\,dx + C_2 = -\frac{1}{2} q_0 x^2 + C_1 x + \underline{C_2}$

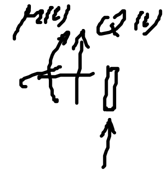
C_1 / C_2 aus RBen: („Kenne ich $M(x)$ oder $Q(x)$ am Rand?")

$x = 0$:



$M(0) = 0$ RB1

$x = L$



$M(L) = 0$ RB2

RBen einsetzen:

RB1: $0 = -\cancel{\frac{1}{2} q_0 0^2} + \cancel{C_1 \cdot 0} + C_2 \Rightarrow C_2 = 0$

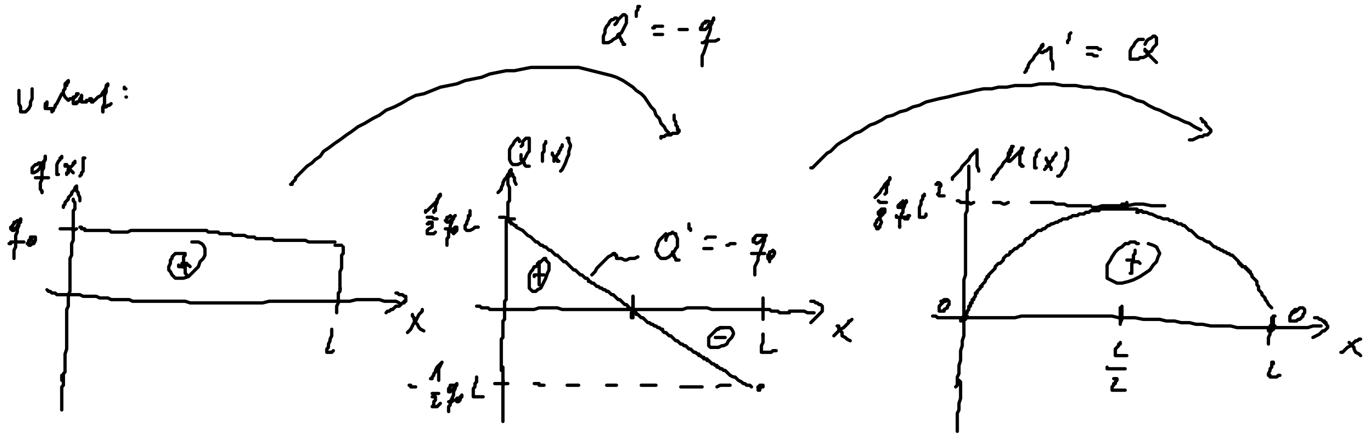
RB2: $0 = -\frac{1}{2} q_0 L^2 + C_1 L \Rightarrow C_1 = \frac{1}{2} q_0 L$

Ergebnis:

$$Q(x) = -q_0 x + \frac{1}{2} q_0 L = q_0 L\left(\frac{1}{2}\cancel{L} - \frac{x}{L}\right) = q_0 L\left(\frac{1}{2} - \left(\frac{x}{L}\right)\right)$$

$$M(x) = -\frac{1}{2} q_0 x^2 + \frac{1}{2} q_0 L x = \frac{1}{2} q_0 L^2\left(\cancel{L}\frac{x}{L} - \frac{x^2}{L^2}\right) = \frac{1}{2} q_0 L^2\left(\left(\frac{x}{L}\right) - \left(\frac{x}{L}\right)^2\right)$$

Verlauf:



$Q(0) = \frac{1}{2} q_0 L$

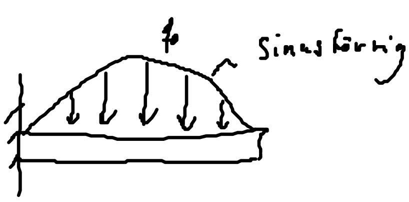
$Q(L) = -\frac{1}{2} q_0 L$

$M(0) = 0$

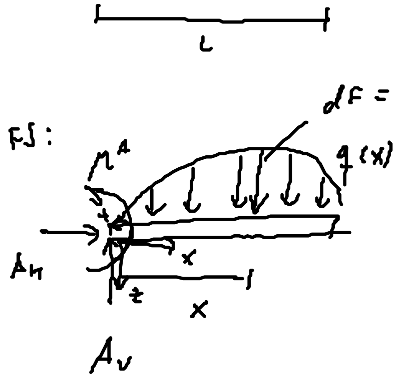
$M(L) = 0$

$M\left(\frac{L}{2}\right) = \frac{1}{8} q_0 L^2$

<u>Sammlung</u>



$$q(x) = q_0 \sin\left(\frac{\pi}{L} x\right)$$



GGB:

$$z:\quad 0 = -A_V + \int_F dF$$

$$A_V = \int_0^L q(x)\,dx = \int_0^L q_0 \sin\left(\frac{\pi}{L}x\right)dx$$

$$A_V = \left[-\frac{L}{\pi}q_0 \cos\left(\frac{\pi}{L}x\right)\right]_0^L$$

$$A_V = -q_0\frac{L}{\pi}\left(\underbrace{\cos(\pi)}_{=-1} - \underbrace{\cos(0)}_{=1}\right) = 2q_0\frac{L}{\pi}$$

$$M^A:\quad 0 = M^A - \int_M dM \Rightarrow M^A = \int_0^L x\,q(x)\,dx$$

$$M^A = -x q_0\frac{L}{\pi}\cos\left(\frac{\pi}{L}x\right)\Big|_0^L + \int_0^L q_0\frac{L}{\pi}\cos\left(\frac{\pi}{L}x\right)dx = q_0\frac{L^2}{\pi}$$

$$\Longrightarrow\quad x_S = \frac{\int_0^L x\,q(x)\,dx}{\int_0^L q(x)\,dx} = \frac{\cancel{q_0}\frac{L^{\cancel{2}}}{\cancel{\pi}}}{2\cancel{q_0}\frac{\cancel{L}}{\cancel{\pi}}} = \frac{1}{2}L$$

Angriffspunkt der Resultierenden