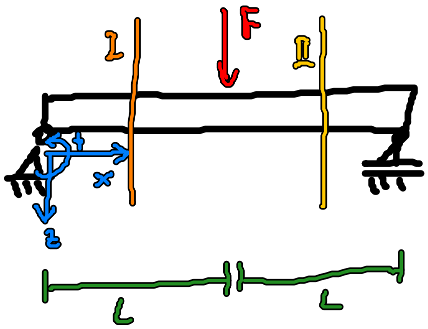
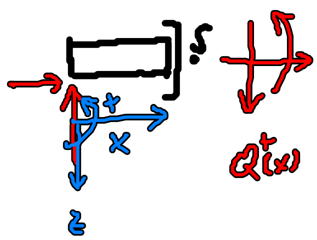
6. Übung – Schnittlasten



positives Ufer:

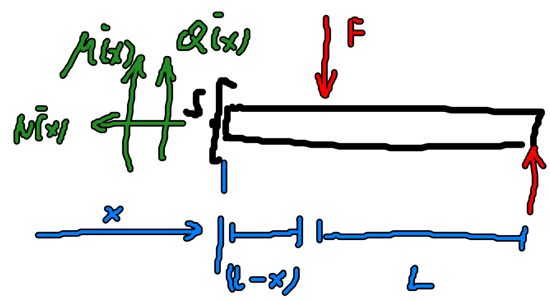


$M^+(x)$ – Biegemoment

$N^+(x)$ – Normalkraft

$Q^+(x)$ – Querkraft

negatives Ufer:

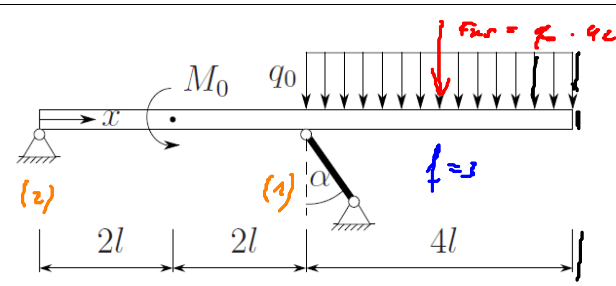


1) Globalschnittverfahren → A62

2) Schnittlast-DGL-Verfahren → A59

Aufgabe 62

. Auf den skizzierten Balken wirkt ein Einzelmoment $M_0 = 4q_0l^2$ und eine konstante Streckenlast q_0. Gesucht sind die Schnittlastenverläufe. Gehen sie zu deren Berechnung wie folgt vor:



(a) Ist das System statisch bestimmt gelagert? Begründen Sie Ihre Antwort.

(b) Bestimmen Sie alle Lagerreaktionen.

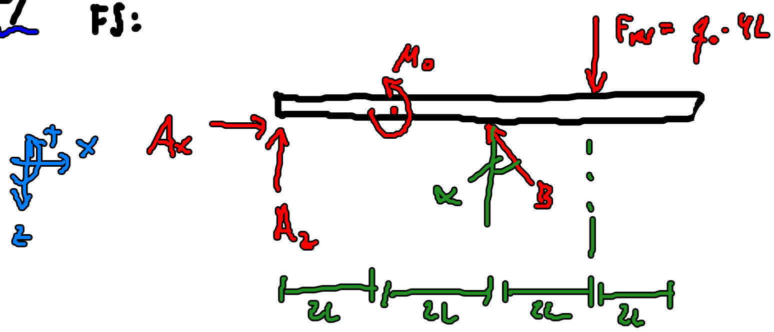
(c) Berechnen Sie die Schnittgrößen $N(x)$, $Q(x)$ und $M(x)$ und skizzieren Sie diese qualitativ unter Angabe markanter Werte (Nullstellen, Extrema etc.).

Geg.: q_0, l, α, $M_0 = 4q_0l^2$

a) • $n = f - r - v \overset{!}{=} 0$ • nicht beweglich o. verspannbar

$n = 3 - 3 - 0 = 0$ ✓

b) FS:



$x: \quad 0 = A_x - B\sin(\alpha) \Rightarrow A_x = B\sin(\alpha) = 5q_0L\frac{\sin(\alpha)}{\cos(\alpha)} = 5q_0L\tan(\alpha)$

$z: \quad 0 = -A_z - B\cos(\alpha) + q_0 4L \Rightarrow A_z = 4q_0L - B\cos(\alpha) = -q_0L$

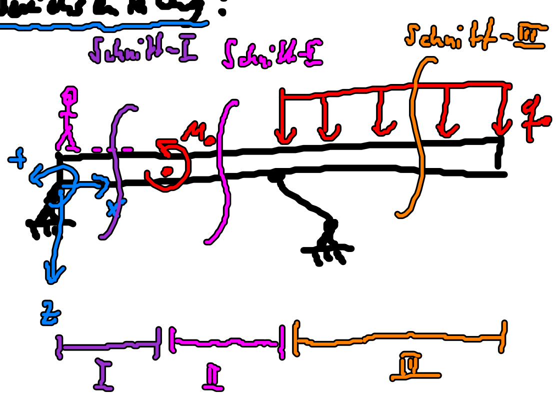
$M^A: \quad 0 = M_0 + B\cos(\alpha)4L - 4q_0L\,6L \Rightarrow B = \frac{1}{\cancel{4L}\cos(\alpha)}\left(-\cancel{M_0} + \cancel{4}q_0L^{\cancel{2}}6\right)$ $\quad 4q_0L^2$

$B = \frac{1}{\cos(\alpha)}\left(-q_0L + 6q_0L\right)$

$B = \frac{5q_0L}{\cos(\alpha)}$

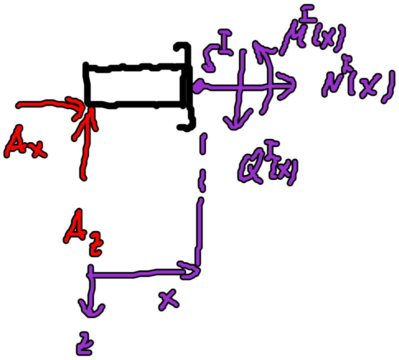
c) Schnittgrößen

1) Bereichseinteilung:



2) Freischnitte

I) $0 < x < 2L$

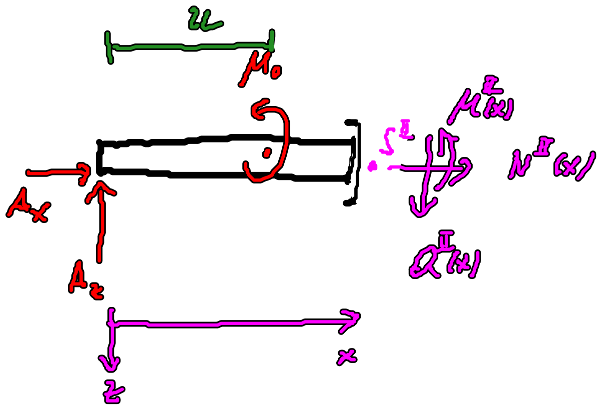


$$x: \quad 0 = A_x + N^I(x) \implies N^I(x) = -A_x = -5q_0 l \tan(\alpha)$$

$$z: \quad 0 = -A_z + Q^I(x) \implies Q^I(x) = A_z = -q_0 l$$

$$M^{S^I}: \quad 0 = M^I(x) - A_z \cdot x \implies M^I(x) = A_z x = -q_0 l x$$

II) $2L < x < 4L$



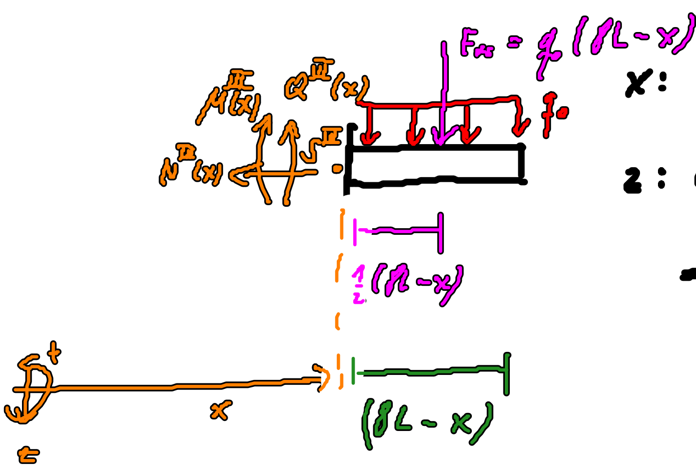
$$x: \quad 0 = A_x + N^{II}(x) \implies N^{II}(x) = -5q_0 l \tan(\alpha)$$

$$z: \quad 0 = -A_z + Q^{II}(x) \implies Q^{II}(x) = -q_0 l$$

$$M^{S^{II}}: \quad 0 = -A_z x + M^{II}(x) + M_0$$

$$\implies M^{II}(x) = -q_0 l x - 4q_0 l^2$$

III) $4L < x < 8L$



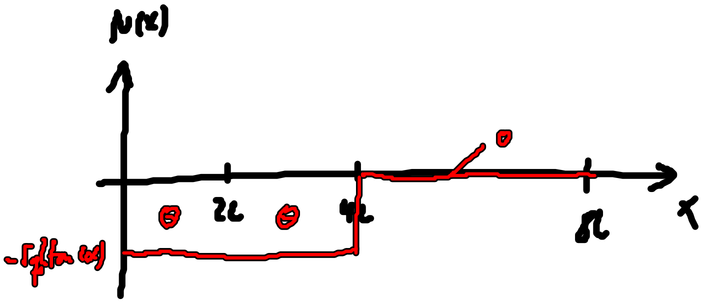
$$x: \quad 0 = -N^{III}(x) \implies N^{III}(x) = 0$$

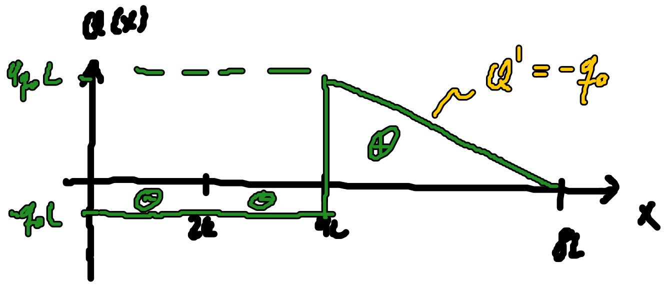
$$z: \quad 0 = -Q^{III}(x) + q_0(8L-x)$$

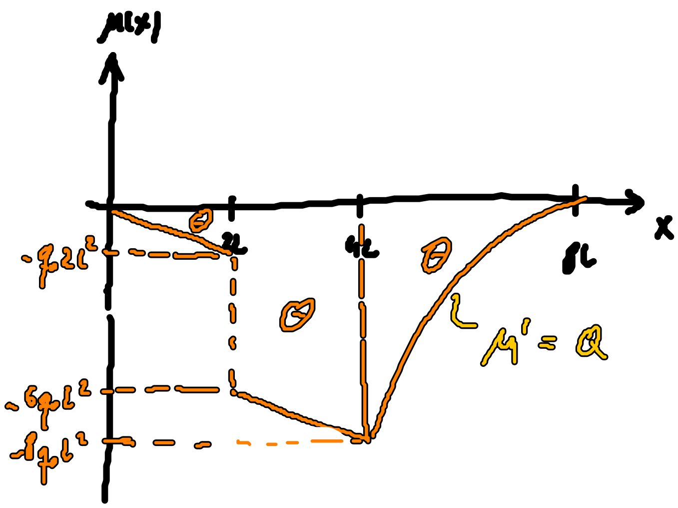
$$\implies Q^{III}(x) = q_0(8L-x)$$

$$M^{S^{III}}: \quad 0 = -M^{III}(x) - q_0(8L-x)\frac{1}{2}(8L-x) \implies M^{III}(x) = -\frac{1}{2} q_0 (8L-x)^2$$

3.) Verlauf







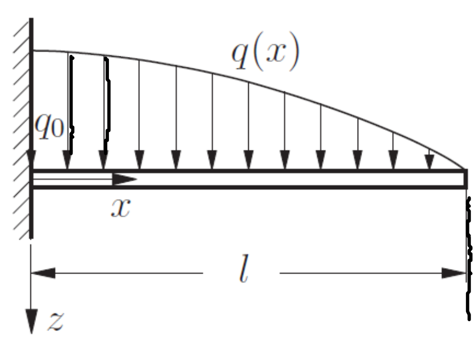
SL-DGL:

$$\frac{dQ}{dx} = -q(x)$$

$$\frac{dM}{dx} = Q(x)$$

Aufgabe 5)

Der skizzierte Balken ist links fest eingespannt und wird durch eine cosinusförmige Streckenlast $q(x)$ belastet.



(a) Berechnen Sie den Verlauf der Schnittgrößen (Biegemoment, Querkraft, ~~Normalkraft~~).

(b) Skizzieren Sie den Verlauf der Schnittgrößen unter Angabe charakteristischer Werte.

(c) Wie groß ist das maximale Biegemoment?

Geg.: q_0, l

Lösung mit SL-DGLen

(1) $\frac{dQ}{dx} = -q(x) \Rightarrow Q(x) = -\int q(x)\,dx + C_1$

(2) $\frac{dM}{dx} = Q(x) \Rightarrow M(x) = \int Q(x)\,dx + C_2$

a) SL-bestimmen

$q(x) = q_0 \cos\left(\frac{\pi}{2L} x\right)$

$\sin(x)' = \cos(x)$

$\cos(x)' = -\sin(x)$

$\sin\left(\frac{\pi}{2L}x\right)' = \frac{\pi}{2L} \cos\left(\frac{\pi}{2L}x\right)$

$(\ldots)^{-1}$

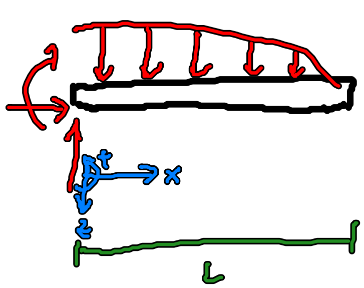
1) Integration:

(1)' $Q(x) = -\int q_0 \cos\left(\frac{\pi}{2L}x\right)dx + C_1 = -q_0 \frac{2L}{\pi} \sin\left(\frac{\pi}{2L}x\right) + C_1$

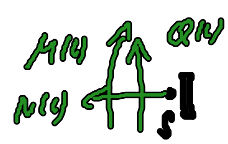
s.o.

(2)' $M(x) = \int\left(-q_0 \frac{2L}{\pi} \sin\left(\frac{\pi}{2L}x\right) + C_1\right)dx + C_2 = +q_0\left(\frac{2L}{\pi}\right)^2 \cos\left(\frac{\pi}{2L}x\right) + C_1 x + C_2$

2) RBen: Wo kenne ich $Q(x)$ oder $M(x)$



$x = L$:

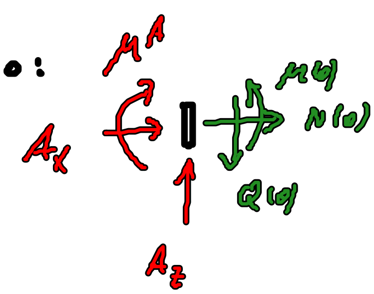


$x: \; 0 = N(L)$

$z: \; 0 = Q(L)$ – RB 1

$M^S: \; 0 = M(L)$ – RB 2

$x = 0$:



$x: \; N(0) = -A_x$?

$z: \; Q(0) = A_z$?

$M^S: \; M(0) = M^A$?

$\Rightarrow$ keine RBen, A_x, A_z, M^A sind nicht bekannt!

3) RBen anwenden:

RB-1: $$0 = -q_0 \frac{2L}{\pi} \underbrace{\sin\left(\frac{\pi}{2L}\cdot L\right)}_{=1} + C_1 \Rightarrow C_1 = q_0 \frac{2L}{\pi}$$

RB-2: $$0 = q_0 \left(\frac{2L}{\pi}\right)^2 \underbrace{\cos\left(\frac{\pi}{2L}\cdot L\right)}_{=0} + C_1 L + C_2 \Rightarrow C_2 = -C_1 L = -q_0 \frac{2L^2}{\pi}$$

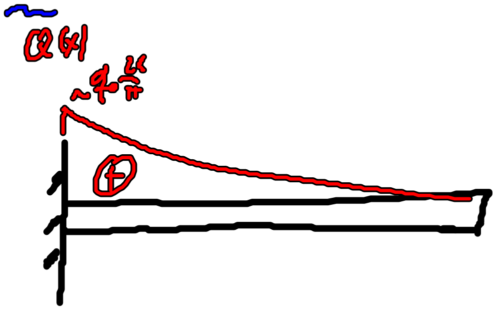
damit:

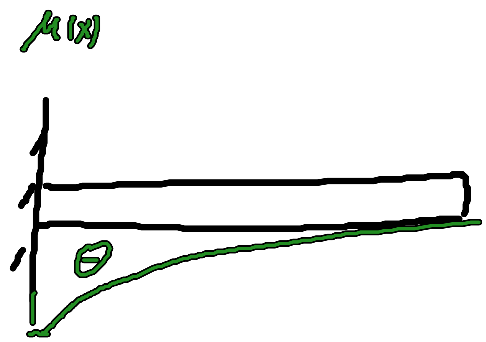
$$Q(x) = -q_0 \frac{2L}{\pi} \sin\left(\frac{\pi}{2L} x\right) + q_0 \frac{2L}{\pi} = q_0 \frac{2L}{\pi}\left(1 - \sin\left(\frac{\pi}{2L} x\right)\right)$$

$$M(x) = q_0 \left(\frac{2L}{\pi}\right)^2 \cos\left(\frac{\pi}{2L} x\right) + q_0 \frac{2L}{\pi} x - q_0 \frac{2L^2}{\pi}$$

$$= q_0 \frac{2L}{\pi}\left(\frac{2L}{\pi} \cos\left(\frac{\pi}{2L} x\right) + x - L\right)$$

b) Verlauf





$$q_0 \frac{2L^2}{\pi}\left(\frac{2}{\pi} - 1\right)$$

$$\left(\frac{2}{3{,}14} - 1\right) < 0$$

c)

$$M_{max} = M(x=0) = q_0 \frac{2L^2}{\pi}\left(\frac{2}{\pi} - 1\right)$$

Tut: 60,67 ; Ha: 66,73