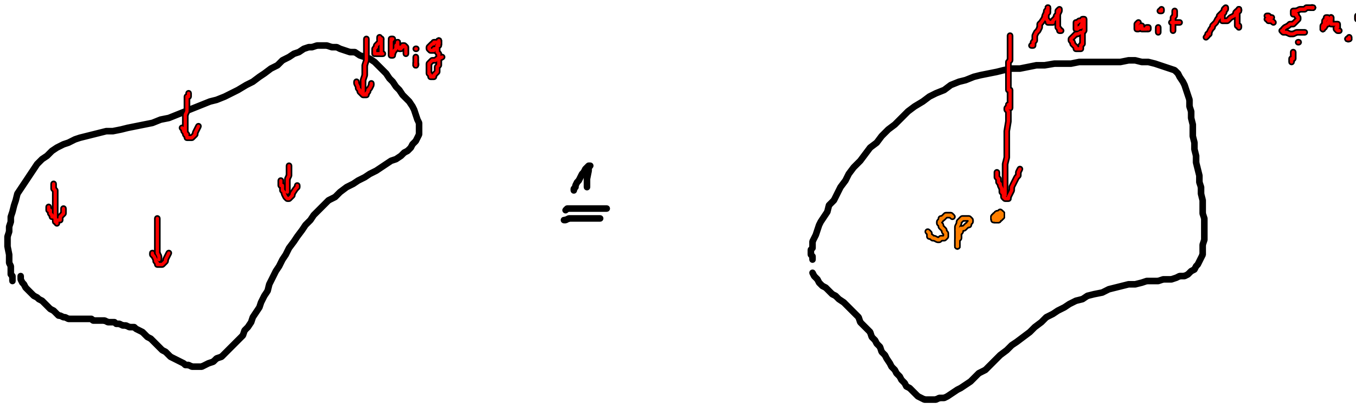
3. Übung - Schwerpunkt

1)



SP: Punkt in dem man sich das eigentlich räumlich verteilte Gewicht konzentriert denken kann!

2)

Einzel-SP$_i$

$$x_s = \frac{\sum_i x_i m_i}{\sum m_i} \quad , \quad \vec{r}_s = \frac{\sum_i \vec{r}_i m_i}{\sum_i m_i}$$

Einzelmassen

Gesamt-SP

3) Kraft-SP:

$$x_s = \frac{\sum_i x_i f_i}{\sum_i f_i} \implies x_s = \frac{\int x f(x)\, dx}{\int f(x)\, dx}$$

Flächen-SP:

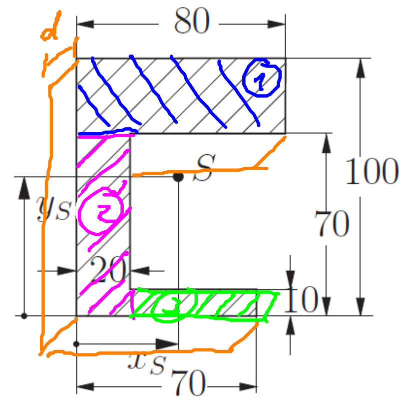
$$x_s = \frac{\sum_i x_i A_i}{\sum_i A_i} \implies x_s = \frac{\int x\, dA}{\int dA}$$

Aufgabe 20

20. Es sind die Schwerpunktabstände x_S und y_S des nebenstehend skizzierten Blechteiles zu bestimmen. (Dicke $d = 3$mm)

homogen $\varrho = $ const.

ges: Schwerpunkt



Massen-SP: $x_S^M = \dfrac{\sum x_i m_i}{\sum m_i}$ mit: $m_i = \varrho V_i = \varrho \cdot D \cdot A_i$

Flächen-SP: $\boxed{x_S^A = \dfrac{\sum x_i A_i}{\sum A_i}}$ $\Longrightarrow$ hier $x_S^M = x_S^A$

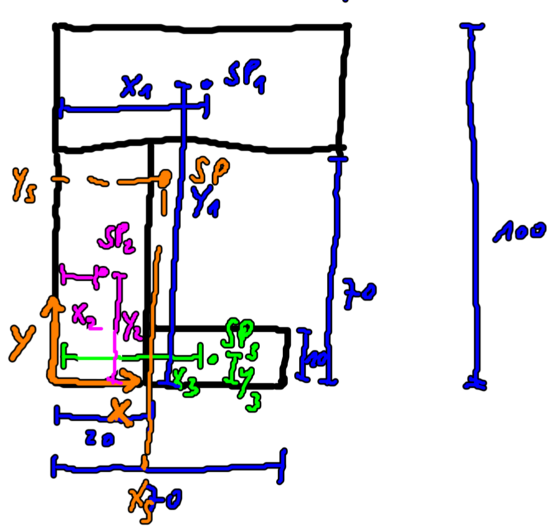
Lösung mit Tabellenverfahren

1) Teile Körper in bekannte Teilstücke

2) Tabelle

i	A_i [mm²]	x_i [mm]	y_i [mm]	$x_i A_i$ [mm³]	$y_i A_i$ [mm³]
1	+2400	40	15+70=85	2400·40 = +96.000	85·2400 = +204.000
2	+1400	10	35	10·1400 = +14.000	35·1400 = +49.000
3	+500	25+20=45	5	45·500 = +22.500	5·500 = +2500
Σ	=4300	–	–	=132.500	=255.500

80



$$A_1 = (80 \cdot 30)\,mm^2$$

$$A_2 = (20 \cdot 70)\,mm^2$$

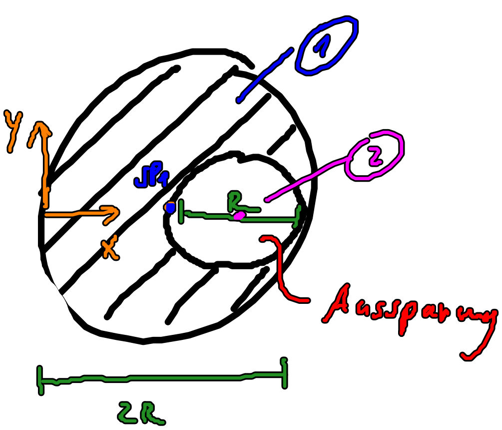
$$A_3 = (50 \cdot 10)\,mm^2$$

damit:

$$x_s = \frac{\sum_i x_i A_i}{\sum_i A_i} = \frac{132.500\ mm^3}{4300\ mm^2} = \underline{\underline{30{,}81\ mm}}$$

$$y_s = \frac{\sum y_i A_i}{\sum A_i} = \frac{255.500\ mm^3}{4300\ mm^2} = \underline{\underline{59{,}42\ mm}}$$

Zusatzaufgabe



geg: R, $\rho = konst.$, $D = konst.$

ges: x_s => Lös mit Tabellenverfahren!

$$x_1 = \frac{2R}{2} = R$$

$$x_2 = \frac{R}{2} + R = \frac{3}{2}R$$

1) Aufteilen

2) $x_s = \dfrac{\sum_i x_i A_i}{\sum_i A_i}$

i	A_i	x_i	$x_i A_i$
1	πR^2	R	πR^3
2	$-\pi\left(\frac{R}{2}\right)^2$	$\frac{3}{2}R$	$-\pi \frac{1}{4} R^2 \frac{3}{2} R = -\frac{3\pi}{8} R^3$
Σ	$\pi R^2 - \frac{1}{4}\pi R^2$		$\pi R^3 - \frac{3\pi}{8} R^3$

!

$$= \frac{3}{4}\pi R^2 \qquad = \pi R^3\left(\frac{8}{8} - \frac{3}{8}\right) = \underline{\frac{5}{8}\pi R^3}$$

$$\Longrightarrow \quad x_s = \frac{\frac{5}{8}\cancel{\pi} R^{\cancel{3}}}{\frac{3}{4}\cancel{\pi}\cancel{R^2}} = \underline{\underline{\frac{5}{6} R}} \qquad \frac{5}{2\cancel{8}} \cdot \frac{\cancel{4}}{3}$$

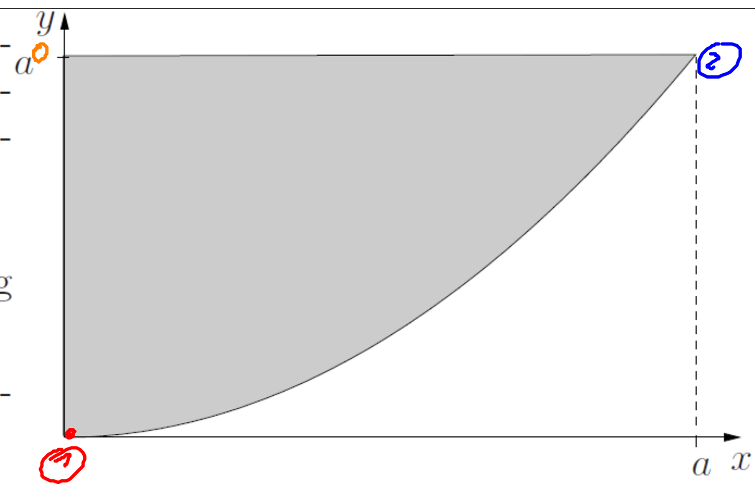
Aufgabe 25

25. Berechnen Sie die Koordinaten des Mittelpunkts der Fläche, die durch den Graphen der Parabel, die y-Achse und die Linie $y = a$ begrenzt wird (s. Skizze).

(a) Stellen Sie die Funktionsgleichung der Parabel auf.

(b) Berechnen Sie alle notwendigen Integrale.

Geg.: a



a) ges: $y(x)$

$$y(x) = Ax^2 + Bx + C \quad \Longrightarrow \quad y = \frac{1}{a}x^2$$

1 $\quad y(x=0) = \frac{1}{a}0^2 = 0$ ✓

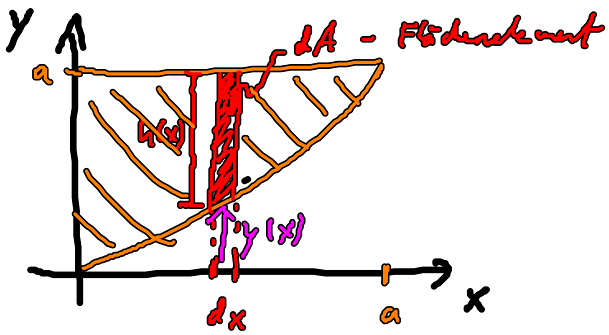
2 $\quad y(x=a) = \frac{1}{a}a^2 = a$ ✓

b) ges: Flächen-SP

$$x_s = \frac{\sum x_i A_i}{\sum A_i} \quad \Longrightarrow \quad x_s = \frac{\int_A x \, dA}{\int_A dA}$$

$$y_s = \frac{\sum y_i A_i}{\sum A_i} \quad \Longrightarrow \quad y_s = \frac{\int_A y \, dA}{\int_A dA}$$

i) x_s!



$A = b \times h$ = Breite × Höhe

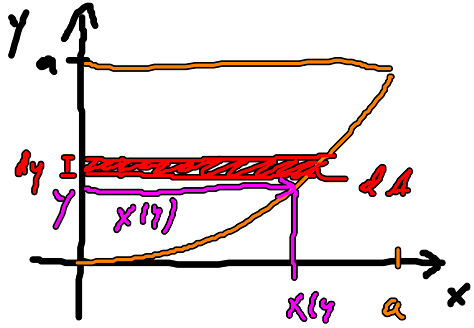
$$\left.\begin{aligned} dA &= h(x)\,dx \\ h(x) &= a - y(x) \end{aligned}\right\} dA = (a - y(x))\,dx$$

$$\Rightarrow \quad x_s = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_0^a x\left(a - \frac{1}{a}x^2\right)dx}{\int_0^a \left(a - \frac{1}{a}x^2\right)dx} = \frac{\int_0^a \left(xa - \frac{1}{a}x^3\right)dx}{\int_0^a \left(a - \frac{1}{a}x^2\right)dx}$$

$$= \frac{\left[\frac{1}{2}ax^2 - \frac{1}{4}\frac{1}{a}x^4\right]_0^a}{\left[ax - \frac{1}{3}\frac{1}{a}x^3\right]_0^a} = \frac{\left(\frac{1}{2}a^3 - \frac{1}{4}\frac{1}{\cancel{a}} \cdot a^{\cancel{4}\,3}\right) - 0}{\left(a^2 - \frac{1}{3}\frac{1}{\cancel{a}}a^{\cancel{3}\,2}\right) - 0}$$

$$= \frac{a^{\cancel{3}}\left(\frac{1}{2} - \frac{1}{4}\right)}{\cancel{a^2}\left(1 - \frac{1}{3}\right)} = \frac{\frac{1}{4}}{\frac{2}{3}}\,a = \frac{3}{8}\,a$$

ii) $$y_s = \frac{\int_A y\,dA}{\int_A dA}$$



$$dA = x(y)\,dy$$

$$y = \frac{1}{a}x^2 \Rightarrow x = \sqrt{ay}$$

$$\left.\begin{array}{l} dA = x(y)\,dy \\ y = \frac{1}{a}x^2 \Rightarrow x = \sqrt{ay} \end{array}\right\} dA = \sqrt{ay}\,dy$$

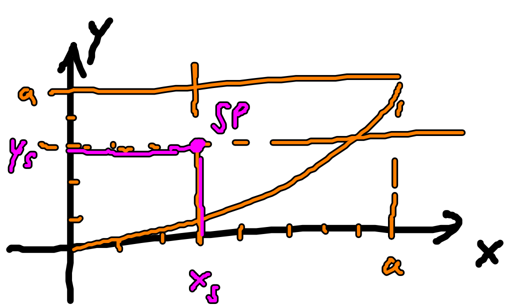
$$\Rightarrow y_S = \frac{\int_A y\,dA}{\int_A dA} = \frac{\int_0^a y\sqrt{ay}\,dy}{\int_0^a \sqrt{ay}\,dy} = \frac{\cancel{\sqrt{a}}\int_0^a \cancel{\sqrt{a}}\,y\,\overbrace{\sqrt{y}}^{y^{\frac{1}{2}}}\,dy}{\cancel{\sqrt{a}}\int_0^a \cancel{\sqrt{a}}\,\sqrt{y}\,dy}$$

$$= \frac{\int_0^a \overbrace{y\cdot y^{\frac{1}{2}}}^{y^{(1+\frac{1}{2})}}\,dy}{\int_0^a y^{\frac{1}{2}}\,dy} = \frac{\int_0^a y^{\frac{3}{2}}\,dy}{\int_0^a y^{\frac{1}{2}}\,dy} = \frac{\left[\frac{2}{5}y^{\frac{5}{2}}\right]_0^a}{\left[\frac{2}{3}\overbrace{y^{\frac{3}{2}}}^{y^{\frac{1}{2}}}\right]_0^a}$$

$$= \frac{\frac{2}{5}a^{\frac{5}{2}} - 0}{\frac{2}{3}a^{\frac{3}{2}} - 0} = \frac{6}{10}\,\frac{a^{\frac{5}{2}}}{a^{\frac{3}{2}}} = \frac{3}{5}a$$

$$a^{\left(\frac{5}{2}-\frac{3}{2}\right)} = a$$

iii) Skizze



Tut: 22a), 29 ; Ha: 22b), 24, 28