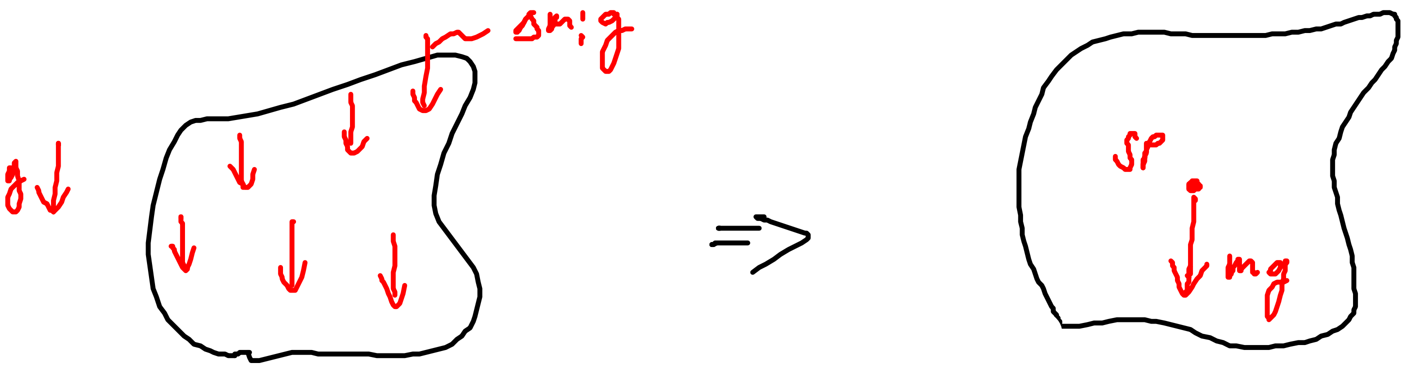
3. Übung – Schwerpunkt

Ue	Tut	Ha
22, 27	24a, 31	24b, 26, 30

Montag keine Spt. bei Robbin!

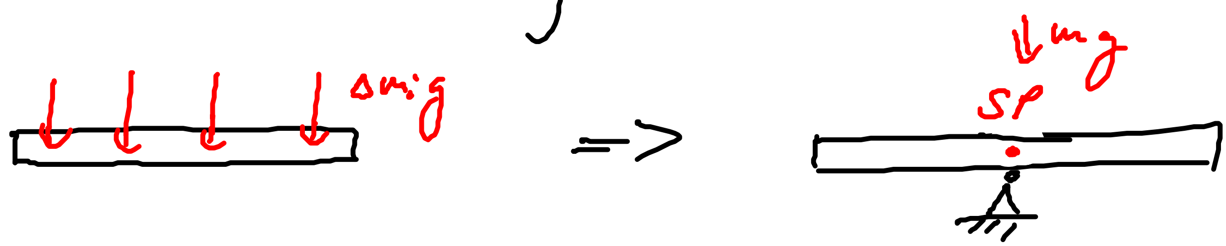
Schwerpunkt:



$m = \sum \Delta m_i$

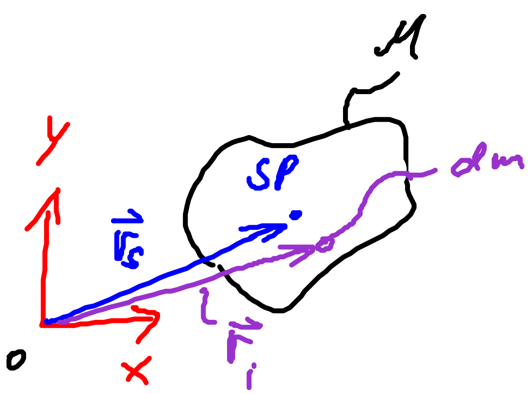
SP: Punkt in dem man sich das eigentlich räumlich verteilte Gewicht eines Körpers konzentriert denken kann – statische Wirkung bleibt unverändert

Bsp.:



Berechnung:

- kontinuierlich:



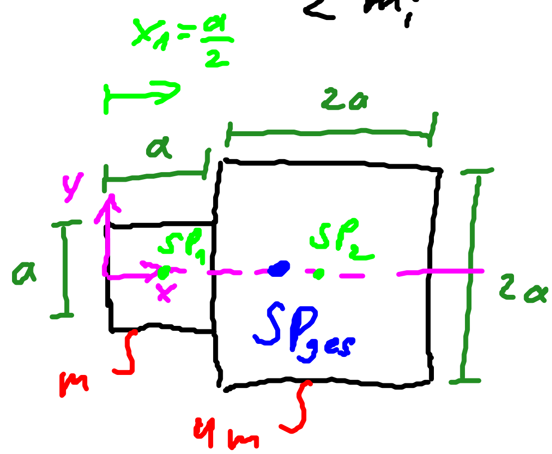
$$\vec{r}_s = \frac{\int_M \vec{r}_i \, dm}{\int_M dm}$$

$$x_s = \frac{\int_m x\,dm}{\int_m dm} \quad ; \quad y_s = \frac{\int_m y\,dm}{\int_m dm}$$

- Diskret:

$$x_s = \frac{\sum x_i m_i}{\sum m_i} \quad ; \quad y_s = \frac{\sum y_i m_i}{\sum m_i}$$

- Beispiel:



$$x_s = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

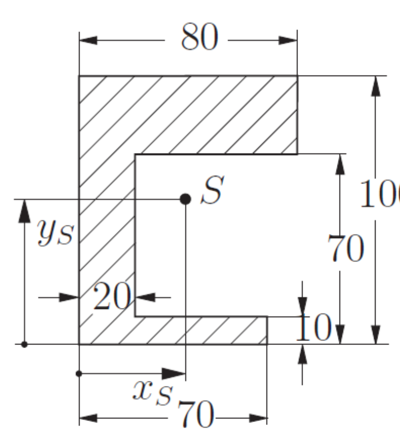
$$= \frac{\frac{a}{2} m + 2a\, 4m}{5m}$$

$$= \frac{am\left(\frac{1}{2} + \frac{16}{2}\right)}{5m}$$

$$x_s = \frac{17}{10}\, a$$

22. Aufgabe:

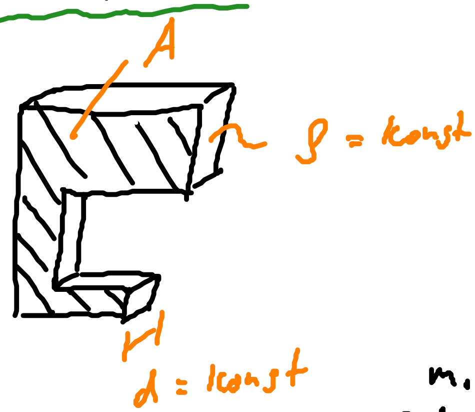
22. Es sind die Schwerpunktabstände x_S und y_S des nebenstehend skizzierten Blechteiles zu bestimmen. (Dicke $d = 3$mm)



Berechnung mittels Tabellenverfahren

$$x_s = \frac{\sum x_i m_i}{\sum m_i} \quad ; \quad y_s = \frac{\sum y_i m_i}{\sum m_i}$$

Sonderfall: Dicke d ist konstant, Dichte ρ ist konstant



$$m = \rho \cdot V = \rho \cdot A \cdot d$$

$$kg = \frac{kg}{m^3} \cdot m^3 = \frac{kg}{m^3} \cdot m^2 \cdot m$$

$$m_i = \rho_i A_i d_i = \rho d A_i$$

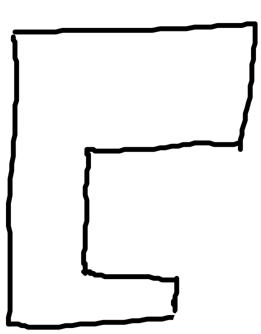
$$\Rightarrow \quad x_S = \frac{\sum x_i \overbrace{\rho_i A_i d_i}^{m_i}}{\sum \rho A_i d_i} = \frac{\cancel{\rho d} \sum x_i A_i}{\cancel{\rho d} \sum A_i} = \frac{\sum x_i A_i}{\sum A_i}$$

$$y_S = \frac{\sum y_i A_i}{\sum A_i}$$

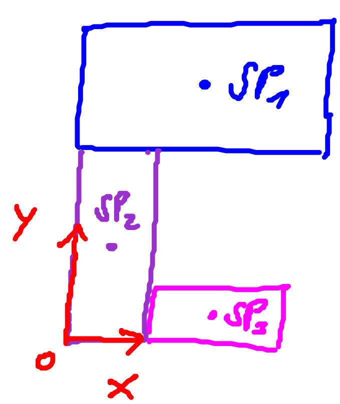
Lösungsweg:

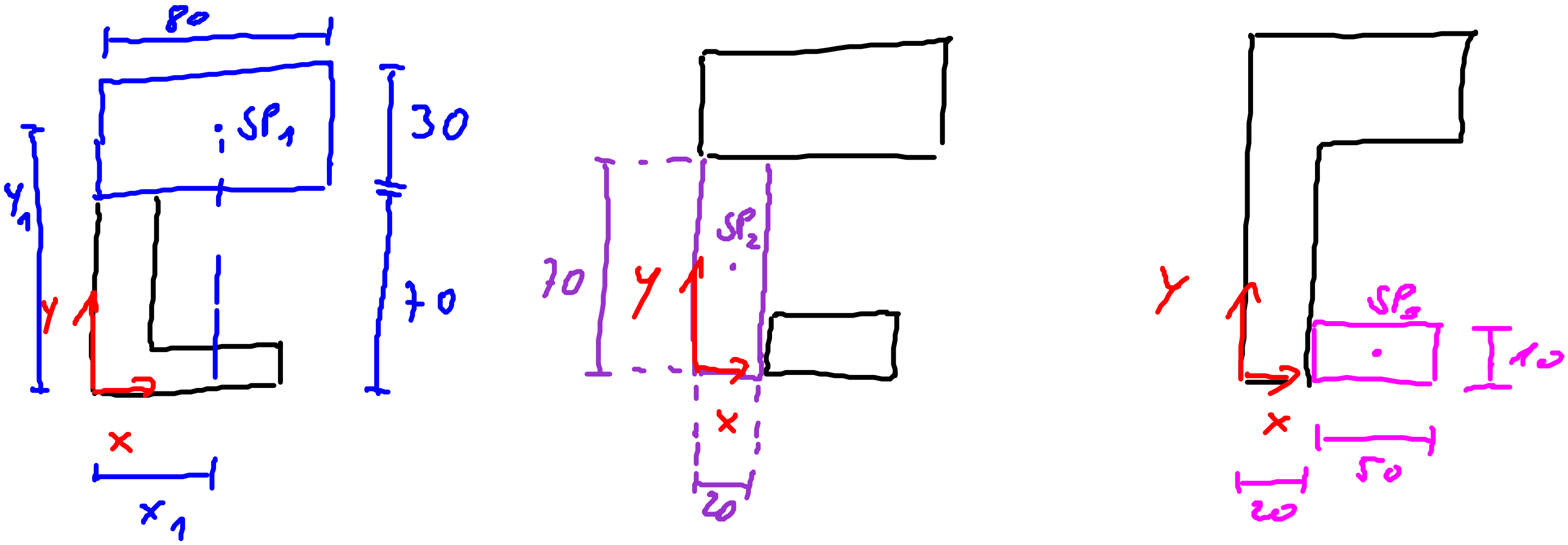
- Zerlegen des Bauteils in bekannte Teilformen
- x_i, y_i, A_i ... ermitteln $\longrightarrow$ in Tabelle
- Berechnung von x_S & y_S

↗ fehleranfällig ↗



$\Rightarrow$





i	x_i	y_i	A_i	$x_i\,A_i$	$y_i\,A_i$
1	40	85	2400	40×2400 = 96 000	85×2400 = 204 000
2	10	35	1400	14 000	49 000
3	45	5	500	22 500	2500
Σ	–	–	4300	132 500	255 500

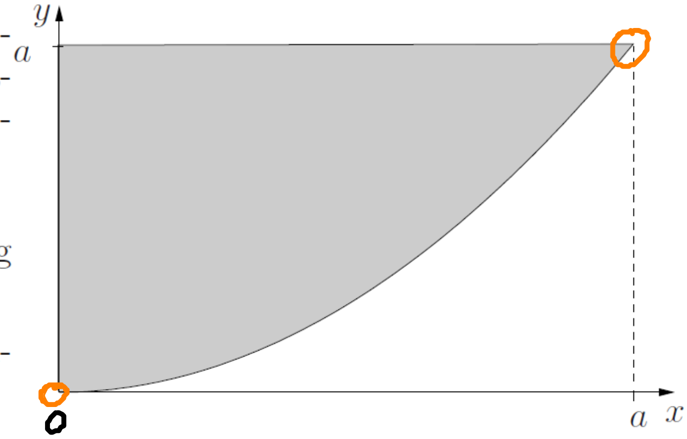
$$x_s = \frac{132\,500}{4300} = 30{,}81\ \text{mm}$$

$$y_s = \frac{255\,500}{4300} = 59{,}42\ \text{mm}$$

27.* Berechnen Sie die Koordinaten des Mittelpunkts der Fläche, die durch den Graphen der Parabel, die y-Achse und die Linie $y = a$ begrenzt wird (s. Skizze).

(a) Stellen Sie die Funktionsgleichung der Parabel auf.

(b) Berechnen Sie alle notwendigen Integrale.



Geg.: a , ges: Flächenschwerpunkt x_s, y_s

a) Funktionsgleichung:

$y(x) = \underline{A} x^2 + \underline{B} x + \underline{C}$ – allg. quadratische Fkt.

3 Unbekannte ⟶ 3 Bedingungen

$y(x=0) = 0 \Rightarrow 0 = A0^2 + B0 + c \Rightarrow C = 0$

$y(x=a) = a \Rightarrow a = Aa^2 + \cancel{B}a \Rightarrow A = \frac{1}{a}$

$y'(x=0) = 0 \Rightarrow 0 = 2A0 + B \Rightarrow B = 0$

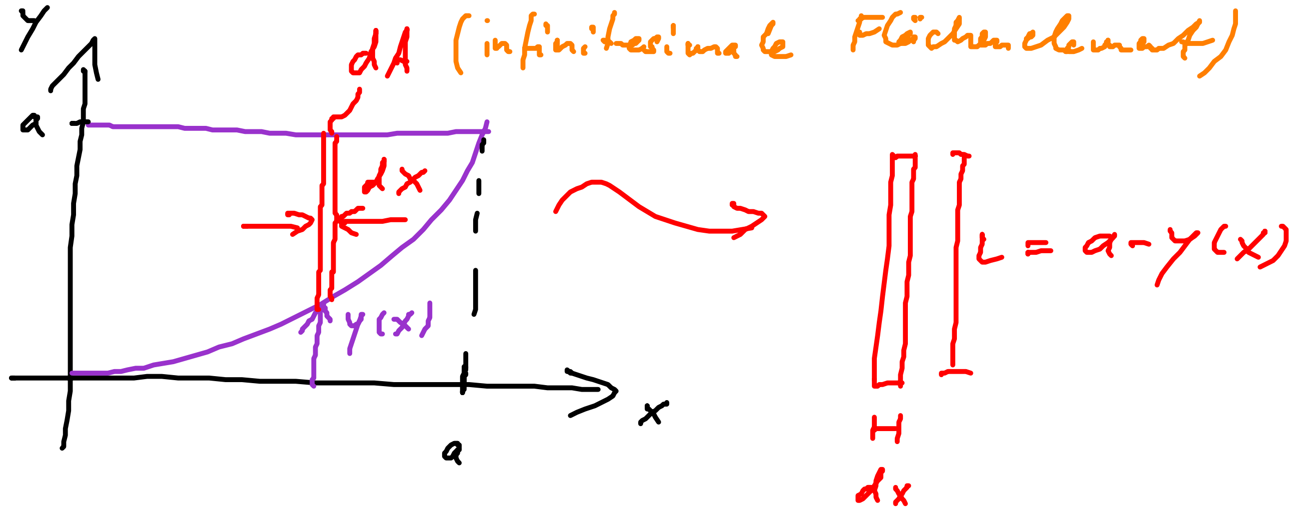
$2Ax + B$

damit: $\boxed{y(x) = \frac{1}{a} x^2}$

b) Berechnen Sie alle notwendigen Integrale:

$x_s = \dfrac{\int_A x \, dA}{\int_A dA}$; $y_s = \dfrac{\int_A y \, dA}{\int_A dA}$

x_s:



$$\Rightarrow \quad dA = (a - y(x))\,dx$$

$$x_s = \frac{\int_0^a x(a - y(x))\,dx}{\int_0^a (a - y(x))\,dx} = \frac{\int_0^a x\left(a - \frac{1}{a}x^2\right)dx}{\int_0^a \left(a - \frac{1}{a}x^2\right)dx}$$

$$= \frac{\int_0^a \left(xa - \frac{1}{a}x^3\right)dx}{\int_0^a \left(a - \frac{1}{a}x^2\right)dx} = \frac{\left[\frac{ax^2}{2} - \frac{x^4}{4a}\right]_0^a}{\left[ax - \frac{x^3}{3a}\right]_0^a}$$

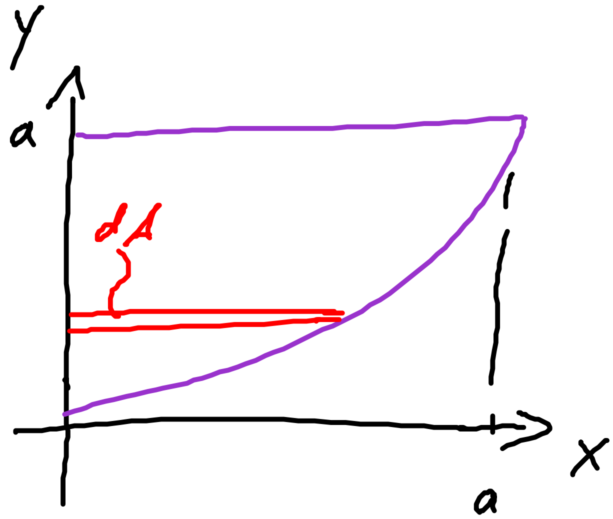
$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

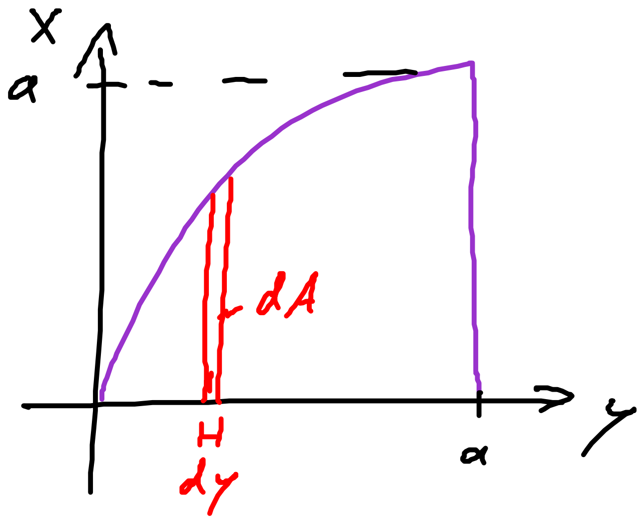
$$= \frac{\frac{a^3}{2} - \frac{a^{\cancel{4}3}}{4\cancel{a}}}{a^2 - \frac{a^{\cancel{3}2}}{3\cancel{a}}} = \frac{\left(\frac{1}{2} - \frac{1}{4}\right)a^{\cancel{3}}}{\left(1 - \frac{1}{3}\right)\cancel{a^2}} \quad \frac{\frac{1}{4}a}{\frac{2}{3}}$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$x_s = \frac{3}{8}a$$

y_s:





$l_y = \underbrace{x(y)}_{\text{Umkehrfunktion!}}$

$$y(x) = \frac{1}{a} x^2 \quad \Big| \cdot a$$

$$y \cdot a = x^2 \quad \Big| \sqrt{\ldots}$$

$$\boxed{x(y) = \sqrt{a y}}$$

$$dA = x(y)\, dy = \sqrt{a y}\, dy$$

$$y_s = \frac{\int_A y\, dA}{\int_A dA} = \frac{\int_0^a y \sqrt{a y}\, dy}{\int_0^a \sqrt{a y}\, dy} =$$

$$= \frac{\int_0^a y\, a^{1/2} y^{1/2}\, dy}{\int_0^a a^{1/2} y^{1/2}\, dy} = \frac{\int_0^a \cancel{a^{1/2}}\, y^{3/2}\, dy}{\int_0^a \cancel{a^{1/2}}\, y^{1/2}\, dy}$$

$$= \frac{\left[\frac{2}{5} y^{5/2}\right]_0^a}{\left[\frac{2}{3} y^{3/2}\right]_0^a} = \frac{\frac{2}{5} a^{5/2}}{\frac{2}{3} a^{3/2}} = \frac{3}{5} \frac{a \cdot a^{3/2}}{a^{3/2}}$$

$$y_s = \frac{3}{5}a$$