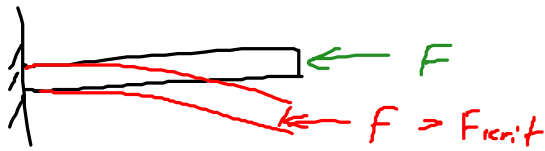


Ke: 151, Tet: 157, 154, Ha: 150, 156

VL-27 - Nachtrag



DGL:  $v''''(x) = -\lambda^2 v''(x)$

$$\lambda^2 = \frac{F}{EI}$$

Aufgabe 1: Stab zwischen 2 Wänden



$\Delta T$  damit Stab knickt?

Querschnitt:



• System entspricht Euler- $\bar{I}$ :  $F_{krit} = EI \left(\frac{\pi}{L}\right)^2$  (1)

$A = \pi a^2$

$I = \frac{\pi}{4} a^4$

• Druckkraft aus thermischer Dehnung:

$$\epsilon_{ges} = \frac{\sigma}{E} + \alpha_T \Delta T \stackrel{!}{=} 0 \text{ (Wände!)}$$

$$\sigma = -E \alpha_T \Delta T \Rightarrow F = \sigma A = -\pi a^2 E \alpha_T \Delta T \text{ (2)}$$

• F muss gerade  $F_{krit}$  sein für Knicken!

(1) in (2):

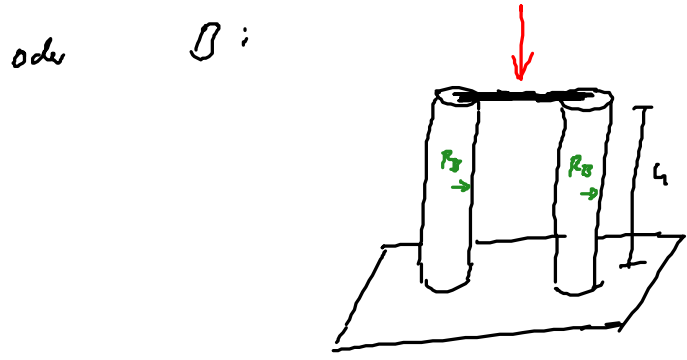
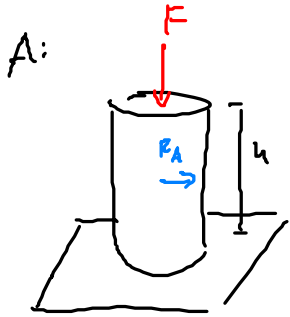
$$EI \left(\frac{\pi}{L}\right)^2 = \pi a^2 E \alpha_T \Delta T$$

$$\cancel{E} \frac{\pi}{4} \cancel{a^4}^2 \left(\frac{\pi}{L}\right)^2 = \cancel{\pi} \cancel{a^2} E \alpha_T \Delta T \Rightarrow \Delta T = \frac{\pi^2}{4 \alpha_T} \left(\frac{a}{L}\right)^2$$

• Werte:  $\frac{L}{a} = 100$ ,  $\alpha_T = 1,2 \cdot 10^{-5} \text{ K}^{-1} \Rightarrow \Delta T = 20 \text{ K}$

II: Ein oder zwei Beine?

F



• Masse soll gleich sein:  $m_A = m_B \Rightarrow \cancel{SV_A} = \cancel{SV_B} \Rightarrow A_A h = A_B h$

$$\cancel{\pi R_A^2} = \cancel{\pi R_B^2} \cdot 2 \Rightarrow R_B = \frac{1}{\sqrt{2}} R_A$$

• Vergleich der kritischen Kraft:

Beide Fällen entsprechen Euler - I:  $F_{krit}^I = \frac{EI}{4} \left(\frac{\pi}{h}\right)^2$

$$A: F_{krit}^A = \frac{EI_A}{4} \left(\frac{\pi}{h}\right)^2 = \frac{E \pi R_A^4}{4 \cdot 4} \left(\frac{\pi}{h}\right)^2 = \frac{E \pi R_A^4 \left(\frac{\pi}{4}\right)^2}{16}$$

$$B: F_{krit}^B = 2 \times F_{krit}^B \text{ - einrechnen} = 2 \cdot \frac{EI_B}{4} \left(\frac{\pi}{h}\right)^2 = 2 \cdot \frac{E \pi R_B^4}{4 \cdot 4} \left(\frac{\pi}{h}\right)^2$$

$$= \frac{E \pi R_A^4 \left(\frac{1}{\sqrt{2}}\right)^4 \frac{1}{4}}{8} \left(\frac{\pi}{h}\right)^2 = \frac{E \pi R_A^4}{4 \cdot 8} \left(\frac{\pi}{h}\right)^2 = \frac{E \pi R_A^4}{32} \left(\frac{\pi}{h}\right)^2$$

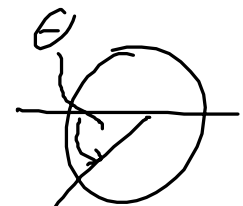
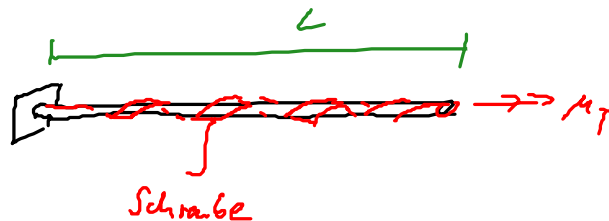
$$\Rightarrow F_{krit}^A = 2 \cdot F_{krit}^B$$

Beispiele!



IV - Andere Instabilitäten:

A) Torsion:

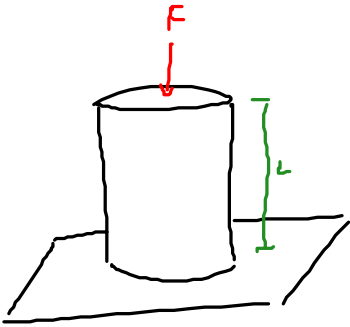


$$\Theta_{\max}^{(1)} = \frac{g E I_y}{G I_p} ; \text{ nach Stab: } I_y = \frac{1}{2} I_p, \quad G = \frac{E}{2(1+\nu)}$$

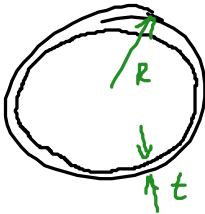
$$\bullet \Theta_{\max} = \frac{g \cancel{I_p} \cancel{2} (1+\nu)}{E \cancel{I_p}} = g(1+\nu)$$

$$\bullet \text{ Stahl: } \nu = 0,3 \Rightarrow \Theta_{\max} = g(1,3) \approx 12 \approx \underline{\underline{4\pi}}$$

B) Dünnwandigkeit  $\Rightarrow$  Linde:



$$F_{\text{krit}} = \frac{E I}{4} \left( \frac{\pi}{L} \right)^2$$

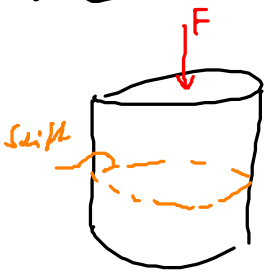


$$I = \pi R^3 t$$

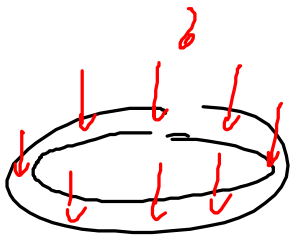
Problem: Lokale Instabilitäten!

Lokales Versagen:  $b_{\text{max}} = \frac{E t}{4R}$  wird  $b_{\text{max}}$  erreicht  $\Rightarrow$  lokales Versagen auf!

Alu + Dose:

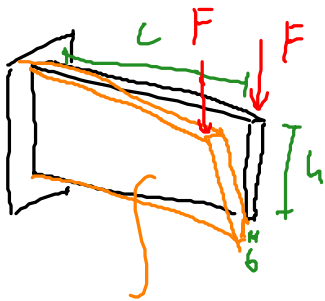


$$\left. \begin{array}{l} E = 70 \text{ GPa} \\ R = 33,5 \text{ mm} \\ t = 0,1 \text{ mm} \end{array} \right\} b_{\text{max}} = 52 \cdot 10^{-3} \text{ GPa}$$



$$\left. \begin{array}{l} b = \frac{F}{A} \\ A = \pi (R^2 - (R-t)^2) = 2\pi R t \end{array} \right\} F_{\text{max}} \approx \underline{\underline{1 \text{ kN}}}$$

c) Stabilität bei Biegung:



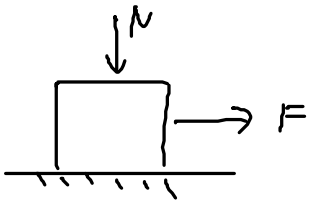
$$w(l) = \frac{1}{3} \frac{F l^3}{EI}$$

$$I = \frac{1}{12} b h^3$$

$$F_{krit} = \frac{26 h^3 E}{l^2 (1+\nu)}$$

VL-28 - Reibung

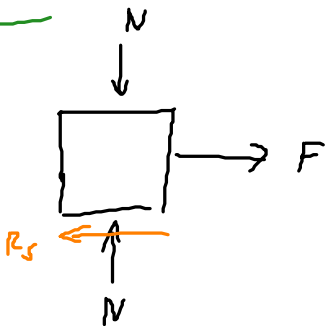
hier: nur trockene Reibung! (Coulombsche Reibung)



I) Gleit- und Haftreibung

Haften:

FS:



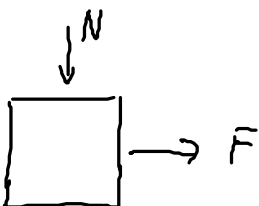
$R_s$  ist proportional zur Normalkraft

$$R_s = \mu_s N$$

Statischer Reibkoeffizient  
(Haftreibungskoeffizient)

Körper bleibt in Ruhe bis  $F = R_s = \mu_s N$

Gleiten:



$$R_k = \mu_k N$$

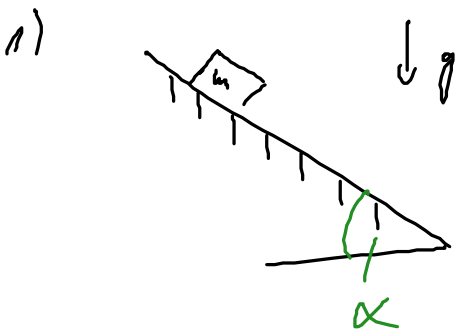
Gleitreibungskoeffizient



Wovon hängt  $\mu$  ab?

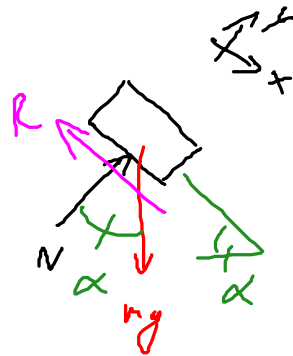
- von vielen Faktoren
- Aber nicht von der Fläche!

II) Reibungswinkel



masse hält gerade hoch! (in Ruhe)

FS:



668:  $\Sigma F_x = 0, -R + mg \sin(\alpha) = 0$  (1)

$\Sigma F_y = 0, N - mg \cos(\alpha) = 0 \Rightarrow N = mg \cos(\alpha)$  (2)

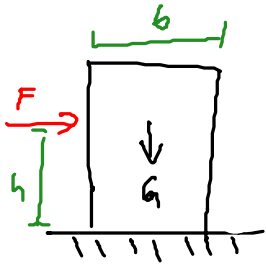
im Grenzfall:  $R = \mu N$  (3) in (1):

$\mu N = mg \sin(\alpha) \Rightarrow \mu \cancel{mg} \cos(\alpha) = \cancel{mg} \sin(\alpha)$   
(2)

$\mu = \tan(\alpha)$

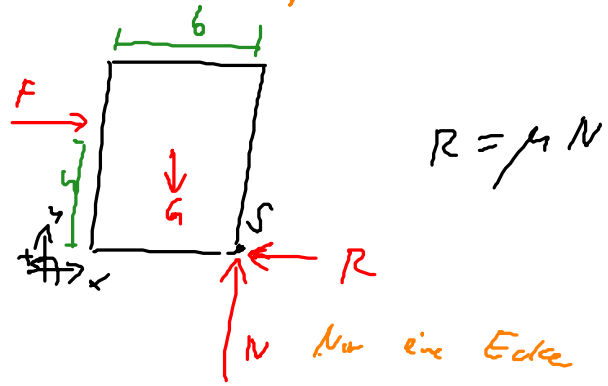
Reibungswinkel

3) Kippende Kiste:



bei welcher Höhe  $h$  kippt die Last? ?

Grenzfall:



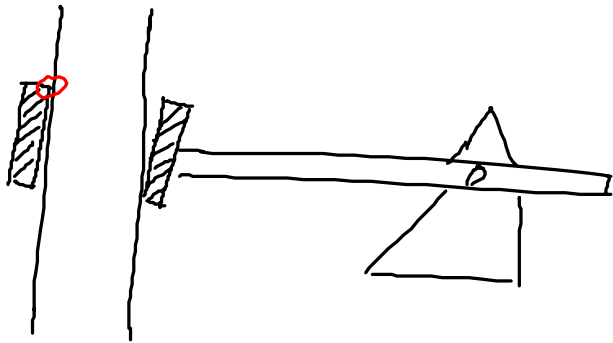
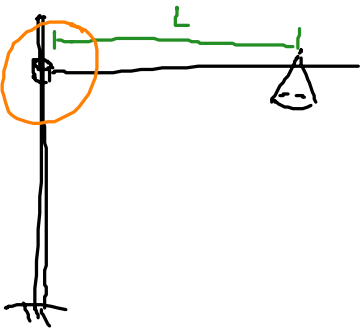
$$\sum F_x \stackrel{!}{=} 0, F - R = 0 \Rightarrow F = R = \mu N = \mu G$$

$$\sum F_y \stackrel{!}{=} 0, -G + N = 0 \Rightarrow N = G$$

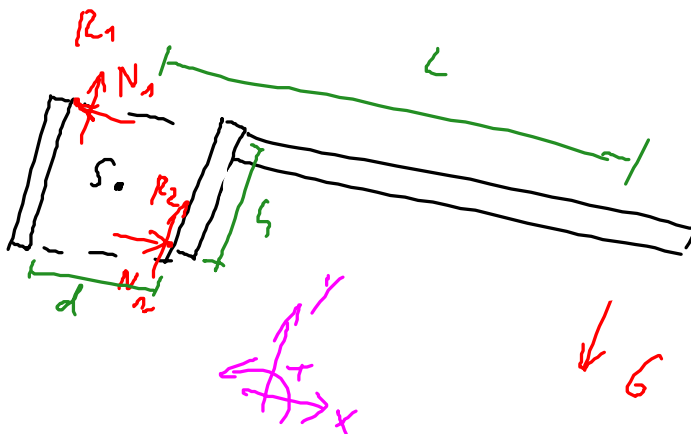
$$\sum M^{(S)} \stackrel{!}{=} 0, -Fh + G \frac{b}{2} = 0$$

$$-\mu G h + G \frac{b}{2} = 0 \Rightarrow h = \frac{b}{2\mu}$$

### iii) Selbstsperrung



FS:



Grenzfall: Körper in Ruhe,  $R = \mu_s N$  }  $R_1 = R_2 = \mu_s N$  (1)

$$\Sigma F_x \stackrel{!}{=} 0, -N_1 + N_2 = 0 \Rightarrow N = N_1 = N_2$$

$$\Sigma F_y \stackrel{!}{=} 0, R_1 + R_2 - G = 0 \Rightarrow R_1 + R_2 = G \quad (2) \quad \text{Falsch!?!}$$

$$\Sigma M^{OX} \stackrel{!}{=} 0, -GL + N_1 \frac{L}{2} + N_2 \frac{L}{2} - \cancel{R_1 \frac{d}{2}} + \cancel{R_2 \frac{d}{2}} = 0$$

$$-GL + NL = 0 \quad (3)$$

$$\cancel{L_c = \frac{L}{2\mu_s} - \frac{d}{2}} \quad \uparrow$$

$$\text{aus (1): } N = \frac{R}{\mu_s}$$

$$\text{aus (2): } G = 2R \Rightarrow R = \frac{G}{2}$$

$$\Rightarrow N = \frac{G}{2\mu_s} \quad \text{in (3):}$$

$$-G \cdot L + \frac{G}{2\mu_s} \cdot L = 0 \Rightarrow L_c = \frac{L}{2\mu_s}$$

1  
kritische Länge!