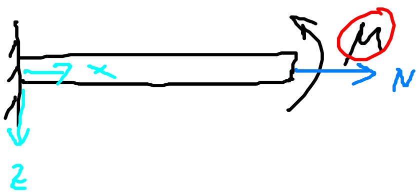
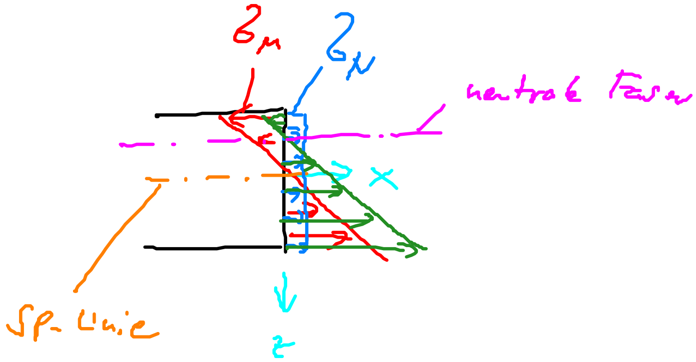
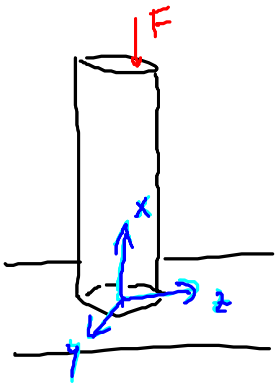
Vorlesung 23

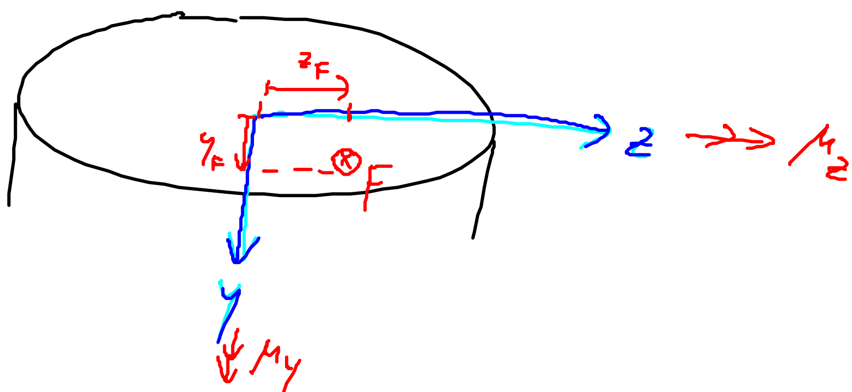
I) Kern des Querschnitts





Säule - exzentrisch belastet:





Spannung: $$\sigma = \frac{N}{A} - \frac{M_z}{I_z} \cdot y + \frac{M_y}{I_y} \cdot z$$

$$\Rightarrow \quad N = -F, \quad M_z = y_F \cdot F, \quad M_y = -z_F \cdot F$$

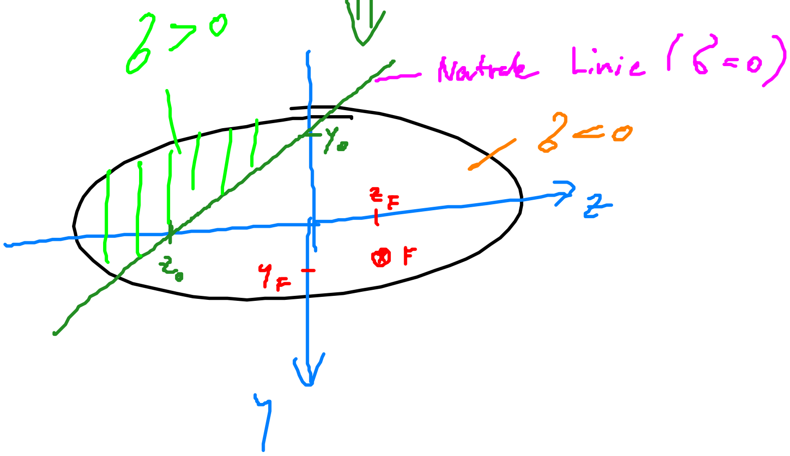
damit: $$\left.\sigma = -\frac{F}{A} - \frac{F}{I_z} \cdot y_F \cdot y - \frac{F}{I_y} \cdot z_F \cdot z \right\} \quad 0 = -\frac{\cancel{F}}{A} - \frac{\cancel{F}}{I_z} \cdot y_F \cdot y - \frac{\cancel{F}}{I_y} \cdot z_F \cdot z$$

Neutrale Fläche: $\sigma \overset{!}{=} 0$

$| \cdot A$

$$\Rightarrow \quad 0 = 1 + \underbrace{\frac{y_F \cdot A}{I_z}}_{-\frac{1}{y_0}} \cdot y + \underbrace{\frac{z_F \cdot A}{I_y}}_{-\frac{1}{z_0}} \cdot z$$

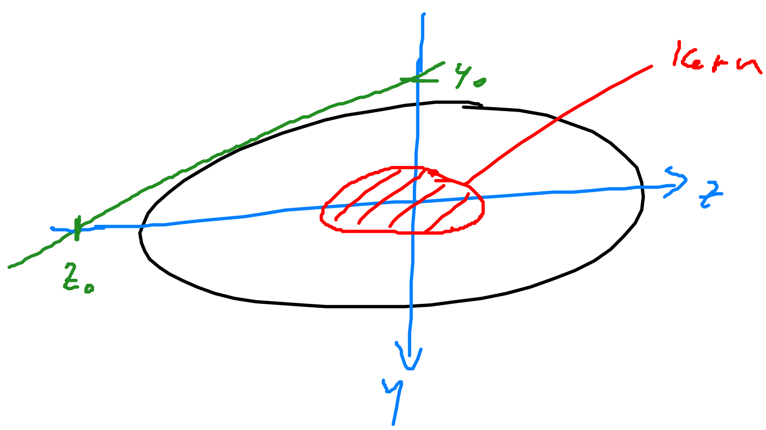
$$\boxed{1 = \frac{y}{y_0} + \frac{z}{z_0}} \Rightarrow \text{Geradengleichung: } y = y_0 - \frac{y_0}{z_0} \cdot z$$



$$y_0 = -\frac{I_z}{A \cdot y_F} \quad ; \quad z_0 = -\frac{I_y}{A \cdot z_F}$$

$\Rightarrow y_0, z_0 < 0$ für $y_F, z_F > 0$

<u>Frage:</u> Wo muss F angreifen, damit $\sigma < 0$ im gesamten Querschnitt? $\Rightarrow$ KERN des Querschnitts

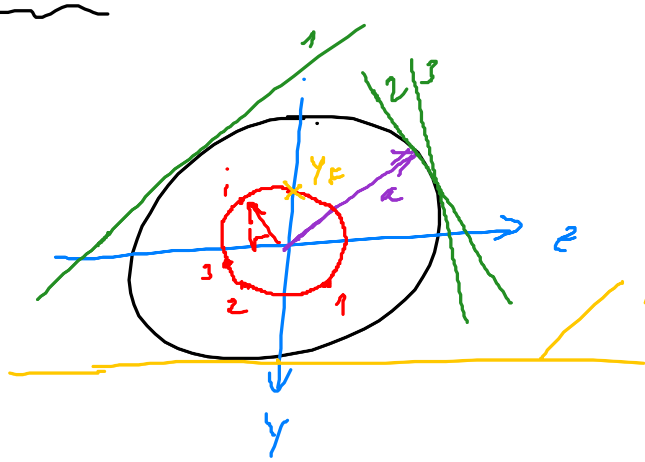


y_F, z_F damit $\sigma < 0$ im gesamten Querschnitt

<u>Beispiel 1:</u> Kreis

$$\frac{y}{y_0} + \frac{z}{z_0} = 1$$

Kern ist kreisförmig wegen Symmetrie



$y \overset{!}{=} a \Rightarrow \frac{a}{y_0} + \frac{z}{z_0} = 1$

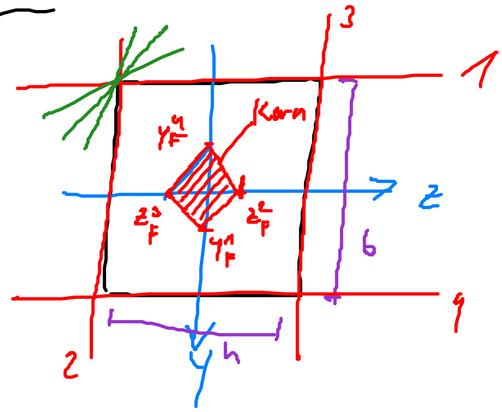
$\Rightarrow z_0 \to \infty$ (Schnittpunkt)

$$\frac{a}{y_0} + \lim_{z_0 \to \infty} \frac{z}{z_0} = 1$$

$$y_0 = a$$

aus Herleitung: $y_F = \frac{-I_z}{A y_0} = \frac{-\frac{\pi}{4}a^4}{\pi a^2 \cdot a} = -\frac{1}{4}a \Rightarrow r = \frac{1}{4}a$

Beispiel 2:

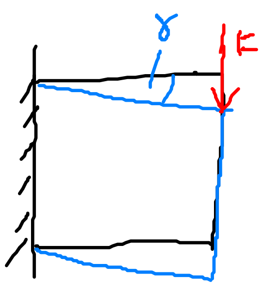


1) $y \overset{!}{=} -\frac{b}{2} \Rightarrow z_0 \to \infty, \; y_0 = -\frac{b}{2}$

damit: $y_F^1 = \frac{-I_z}{A y_0} = \frac{1}{6}h$

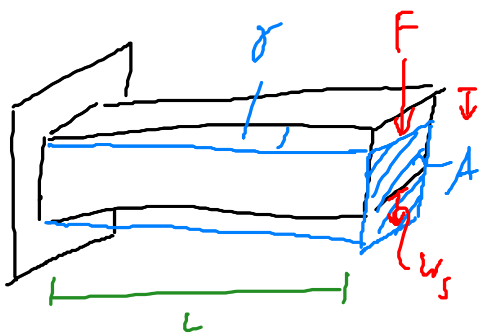
Rest: $z_F^2 = \frac{1}{6}b, \; y_F^4 = -\frac{1}{6}h, \; z_F^3 = -\frac{1}{6}b$

II) Schubeinfluss:



$\tau = G \cdot \gamma$

Schubwinkel



w_s – Durchbiegung durch die Schubspannung

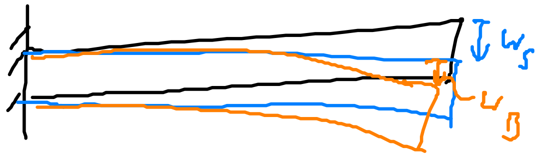
$$\tau = G \cdot \gamma \approx G \cdot \tan\left(\frac{w_s}{L}\right) \approx G \cdot \frac{w_s}{L}$$

$$\boxed{w_s = \frac{\tau L}{G} = \frac{F \cdot L}{G \cdot A}}$$

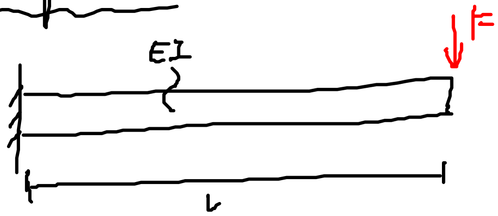
$\tau = \frac{F}{A}$

Überlagerung mit Biegung: $w_{ges} = w_s + w_B = \frac{F \cdot L}{G \cdot A} + w(x=L)$

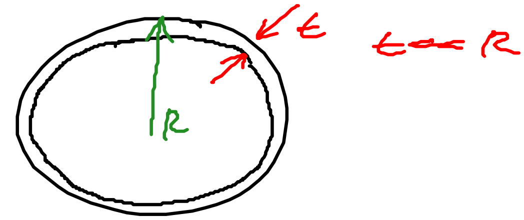
$$\boxed{w_{ges} = \underbrace{\frac{FL}{GA}}_{\text{Schubeinfluss}} + \underbrace{\frac{FL^3}{3EI}}_{\text{Biegeeinfluss}}}$$



Beispiel 3:



ges: $\dfrac{w_S}{w_B}$



$A = 2\pi R t, \quad I_y = \pi R^3 t$

$$\frac{E}{2(1+\nu)} = G$$

$$\frac{w_S}{w_B} = \frac{\frac{FL}{GA}}{\frac{FL^3}{3EI}} = \frac{\cancel{FL}}{GA} \cdot \frac{3EI}{\cancel{F}L^{\cancel{3}2}} = \frac{3\cancel{E}\cancel{\pi}R^{\cancel{3}2}\cancel{t}\,2(1+\nu)}{\cancel{E}\,\cancel{2\pi R t}\cdot L^2} = \frac{\overbrace{3(1+\nu)}^{\approx 4}R^2}{L^2}$$

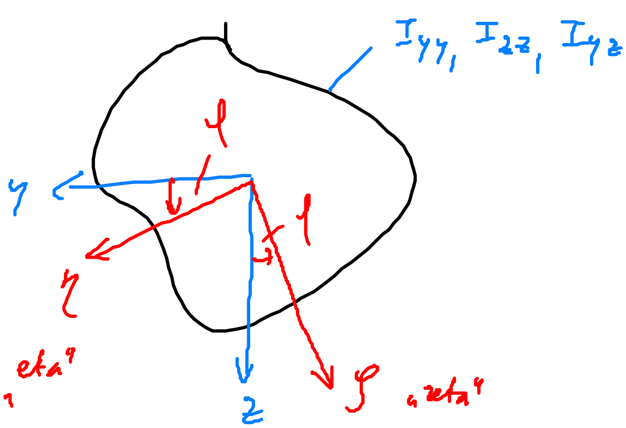
$$\frac{w_S}{w_B} \approx \left(\frac{2R}{L}\right)^2 = \left(\frac{D}{L}\right)^2$$

1) $L = 6R \Rightarrow \dfrac{w_S}{w_B} = \left(\dfrac{2R}{6R}\right)^2 = \dfrac{1}{9} \approx \underline{\underline{10\,\%}}$

2) $L = 20R \Rightarrow \dfrac{w_S}{w_B} = \left(\dfrac{2R}{20R}\right)^2 = \dfrac{1}{100} = \underline{\underline{1\,\%}}$

$\Rightarrow$ Einfluss des Schubes kann ggf. vernachlässigt werden!

Ergänzung zu ÜL-23:

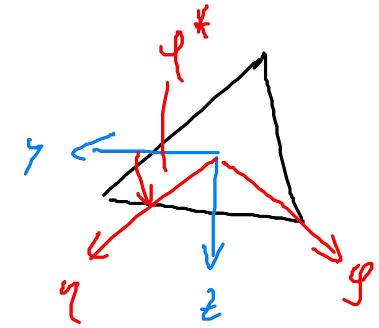


$$I_\eta = \frac{1}{2}(I_y + I_z) + \frac{1}{2}(I_y - I_z)\cos(2\varphi) + I_{yz}\sin(2\varphi)$$

$I_\zeta = \;\ldots$

$I_{\eta\zeta} = \;\ldots$

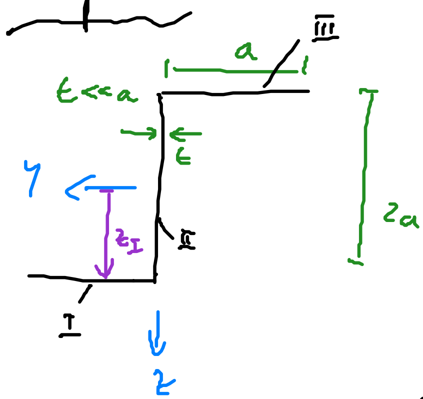
III) Hauptachsensystem : φ^*, so dass $I_{\eta\zeta} \overset{!}{=} 0$



$$\tan(2\varphi^*) = \frac{2 I_{yz}}{(I_y - I_z)}$$

$$I_{1/2} = \frac{1}{2}\left[(I_y + I_z) \pm \sqrt{(I_y - I_z)^2 + 4 I_{yz}^2}\,\right]$$

Beispiel 1: ges: φ^*, $I_{1/2}$



$$I_y = \Sigma I_{y_i} + \Sigma z_{S_i}^2 A_i = \underbrace{2\left(\cancel{\tfrac{1}{12} a t^3}^{\to 0} + a^2 \cdot t a\right)}_{I + III} + \frac{1}{12} t (2a)^3$$

$$I_y = \frac{8}{3} t a^3 \,, \quad I_z = \frac{2}{3} a^3 t$$

$$I_{yz} = \Sigma I_{yz_i} = \overbrace{-\int_0^a y a t \, dy}^{I_{yz_I}} + \overset{I_{yz_{II}}}{0} - \int_{-a}^{0} y(-a) t \, dy = -t a^3$$

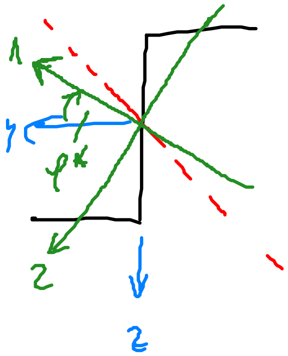
$I_{yz} = -\int yz \, dA$

$\Rightarrow$ In Gleichungen für das Hauptachsensystem:

1) $$\tan(2\varphi^*) = \frac{2 I_{yz}}{(I_y - I_z)} = \frac{-2ta^3}{\frac{8}{3}ta^3 - \frac{2}{3}a^3 t} = -1$$

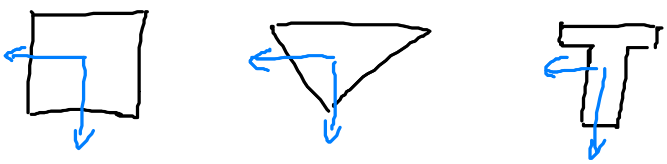
$$\varphi^* = \frac{1}{2}\arctan(-1) = \frac{1}{2}(-45^\circ) = -22{,}5^\circ$$

2) $$I_{1/2} = \frac{1}{2} t a^3 \left(\frac{10}{3} \pm \sqrt{8}\right) \Rightarrow I_1 = 3{,}08\, t a^3 \,, \quad I_2 = 0{,}25\, t a^3$$



$I_1 > I_2 \Rightarrow I_1 \approx 12\, I_2$

IV) Trafo vom Hauptachsensystem ($I_{yz} = 0$)

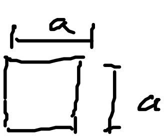


$$I_\eta = \frac{1}{2}(I_y + I_z) + \frac{1}{2}(I_y - I_z)\cos(2\varphi) \qquad (1)$$

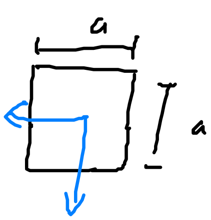
$$I_\zeta = \frac{1}{2}(I_y + I_z) - \frac{1}{2}(I_y - I_z)\cos(2\varphi) \qquad (2)$$

$$I_{\eta\zeta} = -\frac{1}{2}(I_y - I_z)\sin(2\varphi) \qquad (3)$$

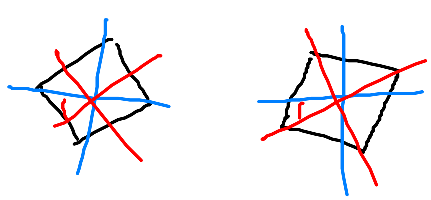
Fall: $I_y = I_z = I$ (beide Hauptachsen-FTM gleich)



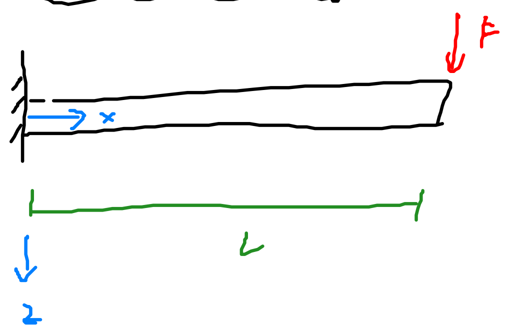
$$\left.\begin{aligned} I_\eta &= \frac{1}{2}\, 2I + 0 = I \\ I_\zeta &= \frac{1}{2}\, 2I + 0 = I \end{aligned}\right\}$$ Jedes um beliebige φ gedrehtes System ist gleich

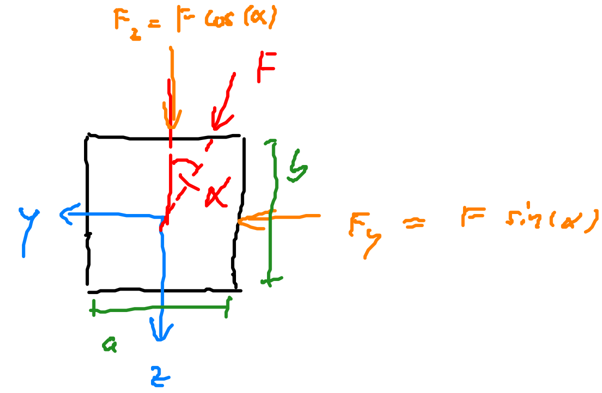


$\hat{=}$



V) Schiefe Biegung:





ges: w_{ges}

$$w_{ges} = \left(w_z^2 + w_y^2\right)^{1/2} =$$

$$\left.\begin{aligned} w_y &= \frac{F_z l^3}{3EI_y} \\ w_z &= \frac{F_y l^3}{3EI_z} \end{aligned}\right\} \quad \vec{w} = \begin{pmatrix} w_y \\ w_z \end{pmatrix} = \frac{1}{3} F \frac{l^3}{E} \frac{12}{ab} \left(\frac{\sin(\alpha)}{a^2} , \frac{\cos(\alpha)}{b^2} \right)$$

$$w_{ges} = |\vec{w}_{ges}| = 4F \frac{l^3}{Eab} \sqrt{\frac{\sin^2(\alpha)}{a^4} + \frac{\cos^2(\alpha)}{b^4}}$$

$\Rightarrow$ Trafo auf 1, 2 System $\Rightarrow$ $\hat{w}$ berechnen $\Rightarrow$ Rück-Trafo