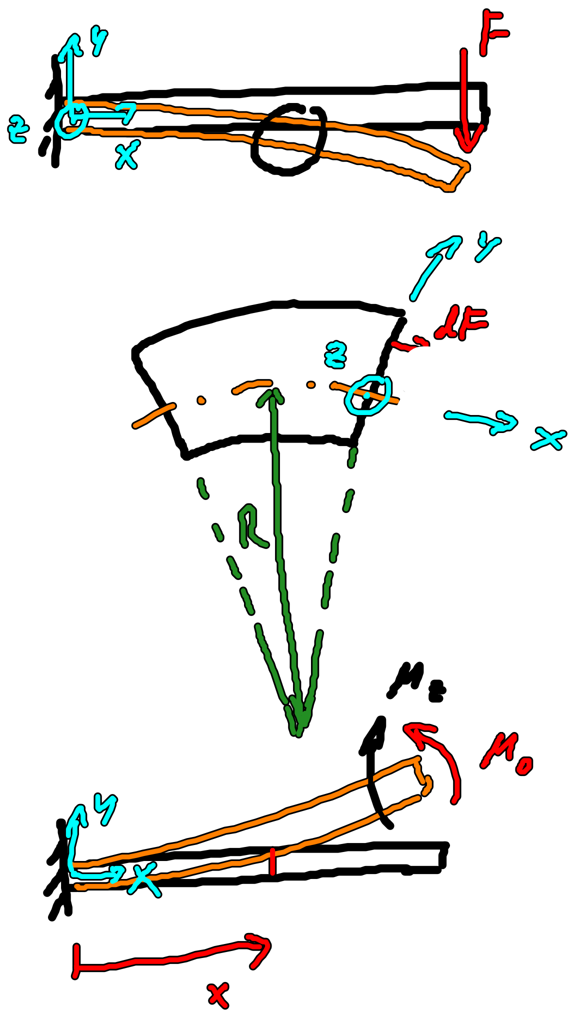
VL - 22 : Spannungsverteilung bei Biegung

I) Spannungsverteilung :

VL - 17:



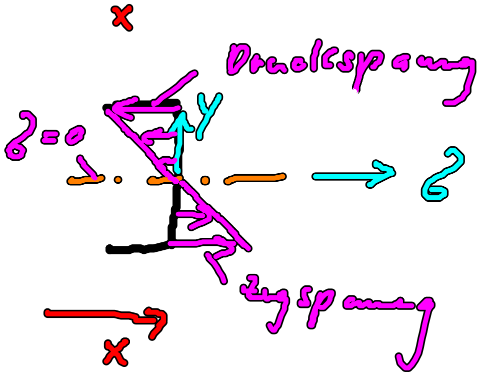
$$\sigma = E\varepsilon = E\frac{y}{R}$$

$$M_F = -\int y\,dF = -\frac{EI_z}{R}$$

$$\Rightarrow \boxed{\sigma = -\frac{M_z}{I_z}\cdot y}$$

$$M_z = M_0 \Rightarrow \sigma = -\frac{M_0}{I_z}\cdot y$$

i)



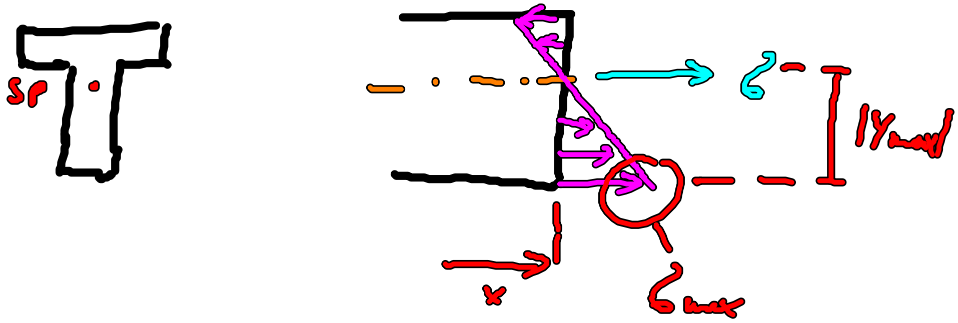
maximale Spannung:

$$\left[\sigma_{max} = \left|\frac{M_{max}}{I_z}\, y_{max}\right| = \frac{|M_{max}|}{W}\right]$$

$$\boxed{\frac{I_z}{|y_{max}|} = W}$$

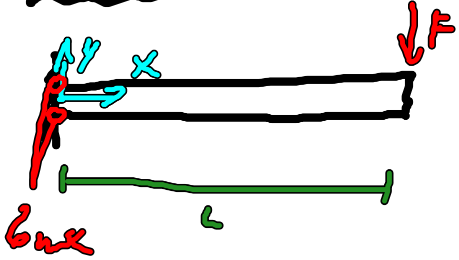
Widerstandsmoment

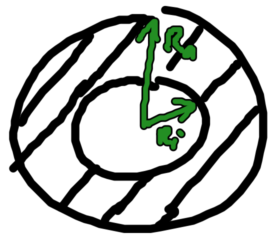
ii)



maximale Spannung am äußeren Rand

Beispiel 1





$R_a = 5\,cm$

$R_i = 4\,cm$

$L = 3\,m$

ges: F damit $\sigma \leq \sigma_{zul} = 150\,MPa$

$$\left\{ \sigma = \frac{M}{I} y \;,\; \sigma_{max} = \frac{|M_{max}|}{W} \;,\; W = \frac{I_z}{|y_{max}|} \right\}$$

$$\frac{|M_{max}|}{W} \leq \sigma_{zul} \;,\quad \text{mit: } |M_{max}| = F \cdot L \;,\; |y_{max}| = R_a$$

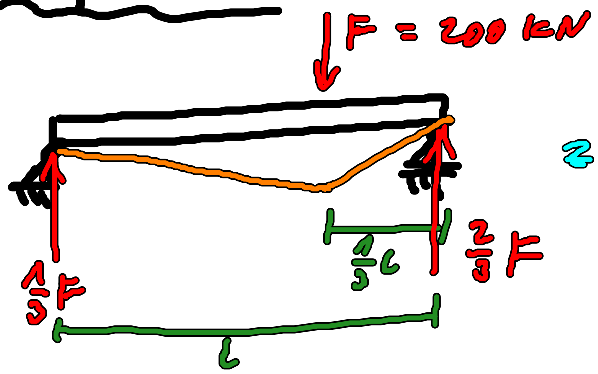
$$W = \frac{I_z}{|y_{max}|} = \frac{\frac{\pi}{4}(R_a^4 - R_i^4)}{R_a}$$

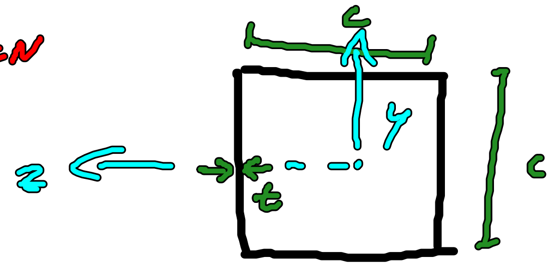
$$\Rightarrow \frac{F \cdot L \cdot 4 \cdot R_a}{\pi (R_a^4 - R_i^4)} \leq \sigma_{zul} \Rightarrow F \leq \frac{\sigma_{zul}\, \pi (R_a^4 - R_i^4)}{4L \cdot R_a}$$

mit Zahlenwerten:

$$\underline{\underline{F \leq 2{,}9 \cdot 10^3\, N}}$$

Beispiel 2:





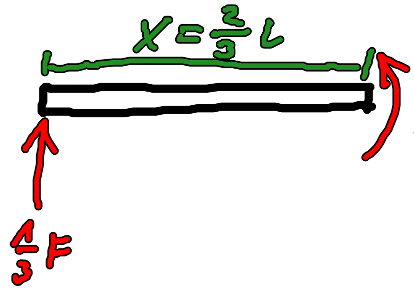
$t << c$

$t = 15\,mm$

$l = 10\,m$

ges: c damit $\sigma_{max} \leq \sigma_{zul} = 200\,MPa$

$$\sigma_{max} = \frac{|M_{max}|}{W} \quad (1)$$



$$M_z = M_{max} = \frac{2}{9} F l \quad (2)$$

$$M_{max} = ?$$

$$W = \frac{I}{|y_{max}|} = \frac{I_a - I_i}{\frac{c}{2}} = \frac{2\left(\frac{1}{12} c^4 - \frac{1}{12}(c-2t)^4\right)}{c}$$

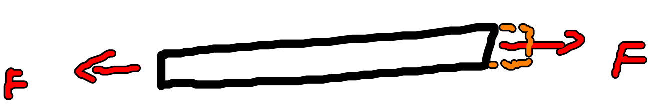
$t << c$

$$= \frac{\left(\cancel{c^4} - \cancel{c^4} + 8c^3 t - \cancel{24 c^2 t^2}_{\to 0} + \cancel{32 c t^3}_{\to 0} - \cancel{16 t^4}_{\to 0}\right)}{6c} = \frac{4}{3} c^2 t \quad (3)$$

(2), (3) in (1):

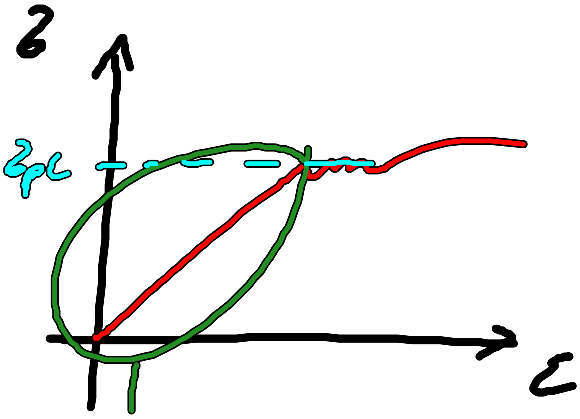
$$\frac{\frac{2}{9} F l}{\frac{4}{3} c^2 t} \leq \sigma_{zul} \Rightarrow c \geq \left(\frac{F l}{6 t \sigma_{zul}}\right)^{1/2} = 0{,}333\,m$$

II) Versagen beim Biegen:



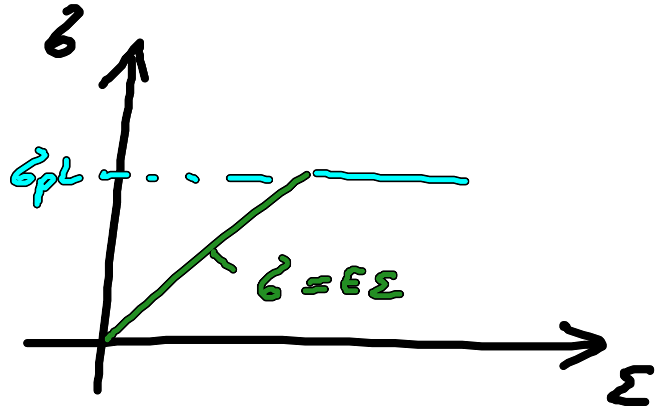
$$\varepsilon = \frac{\Delta l}{l}$$

Spannungs-Dehnungsdiagramm



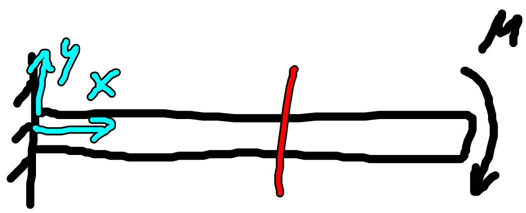
$\sigma = E\varepsilon$ (Hooke)

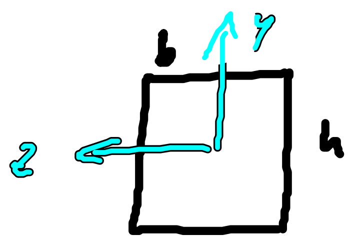
Näherung →

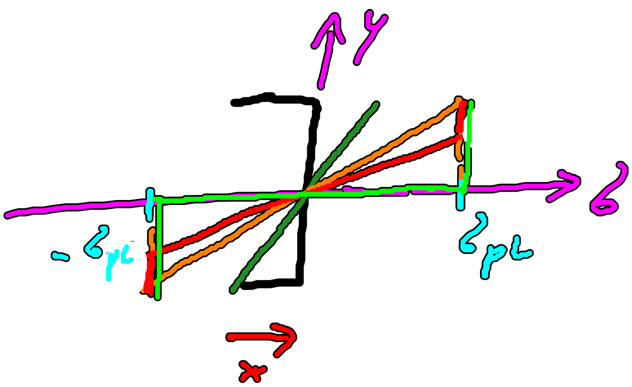


ist σ_{pl} erreicht, ändert sich σ nicht mehr mit ε

Balken:



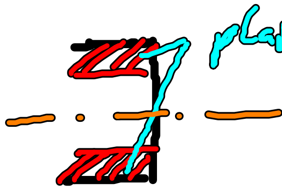




$\sigma_{max} = \frac{M}{W}$

$\sigma_{max} = \sigma_{pL}$

im Außenbereich ist $\sigma = \sigma_{pL}$

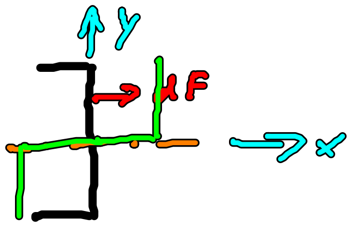


plastisch verformt

komplett plastisch verformt!

→ Wann tritt Versagen auf?

1) plastisches Versagen



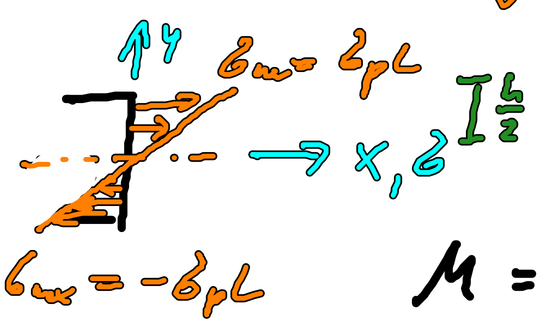
$$dM = -y\,dF = -y\,\sigma\,dA = -y\,\sigma\,b\,dy$$

$$M = \int_{-\frac{h}{2}}^{\frac{h}{2}} -y\,\sigma\,b\,dy = \int_{0}^{\frac{h}{2}} -y\,\sigma_{pL}\,b\,dy + \int_{-\frac{h}{2}}^{0} -y(-\sigma_{pL})\,b\,dy$$

$$M = \sigma_{pL}\,b\left(\left[-\frac{1}{2}y^2\right]_0^{\frac{h}{2}} + \left[\frac{1}{2}y^2\right]_{-\frac{h}{2}}^{0}\right) = -b\,\sigma_p\,\frac{h^2}{4}$$

$$\Longrightarrow \boxed{M_{pL} = \frac{bh^2}{4}\,\sigma_{pL}} \Longrightarrow \sigma = \sigma_{pL}$$

2) elastisches Versagen



$$\sigma = \frac{\sigma_{pL}}{\frac{h}{2}}\,y = \frac{2\sigma_{pL}}{h}\,y$$

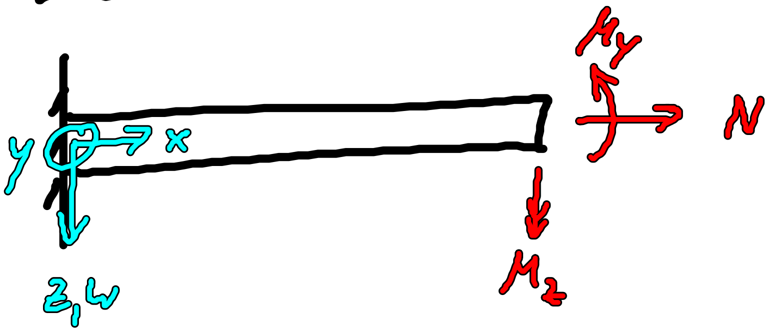
$$M = \int_{-\frac{h}{2}}^{\frac{h}{2}} -y\,b\,\sigma\,dy = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{-2\sigma_{pL}}{h}\,b\,y^2\,dy$$

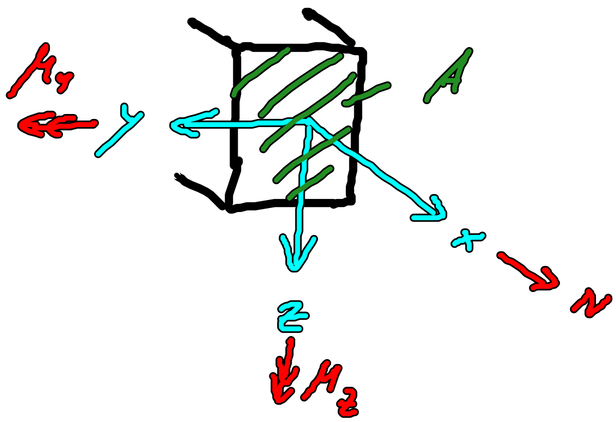
$$M = \left[\frac{-2}{3}\,\frac{\sigma_{pL}}{h}\,b\,y^3\right]_{-\frac{h}{2}}^{\frac{h}{2}} = -\frac{1}{6}\,b\,\sigma_{pL}\,h^2$$

$$\Longrightarrow \boxed{M_{el} = \frac{bh^2}{6}\,\sigma_{pL}} \Longrightarrow \text{In der Randfaser } \sigma = \sigma_{pL}$$

Vergleich: $\dfrac{M_{pL}}{M_{el}} = \dfrac{\frac{1}{4}}{\frac{1}{6}} = \dfrac{3}{2} = 1{,}5$

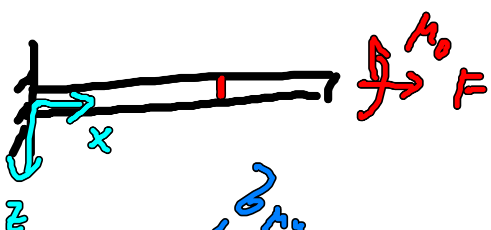
II) Überlagerung von Belastungen

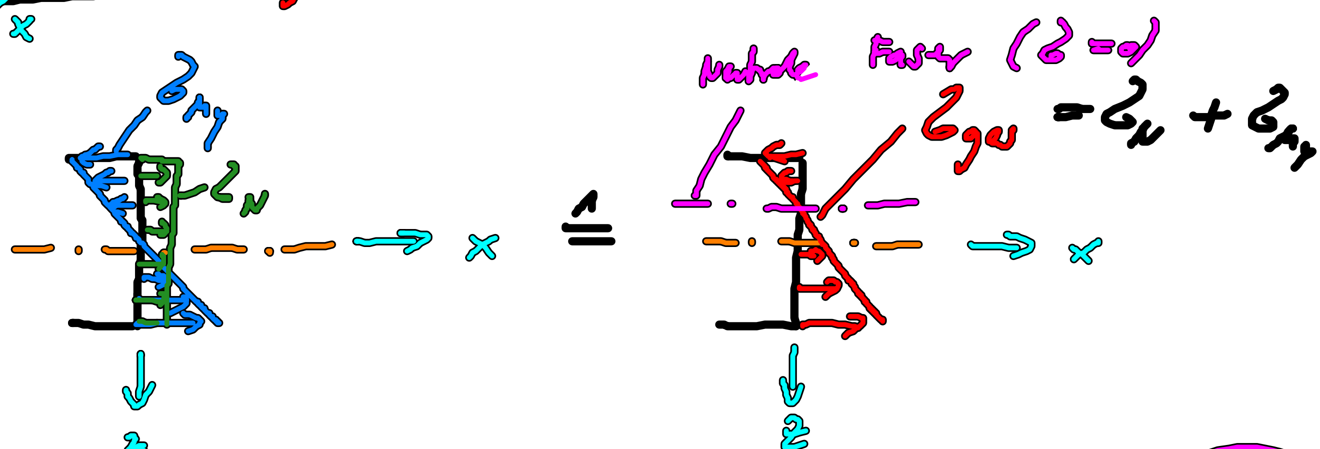




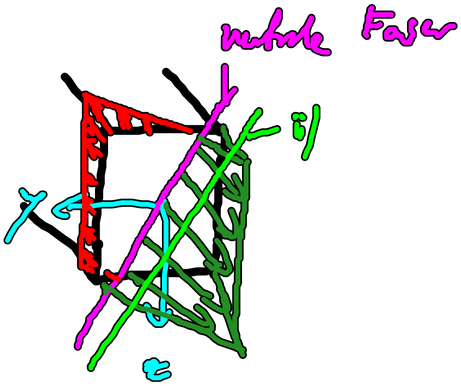
$$\left.\begin{aligned} \sigma_N &= \frac{N}{A} \\ \sigma_{M_z} &= -\frac{M_z}{I_z}\cdot y \\ \sigma_{M_y} &= \frac{M_y}{I_y}\cdot z \end{aligned}\right\} \quad \sigma = \sigma_N + \sigma_{M_z} + \sigma_{M_y}$$

$$\boxed{\sigma = \frac{N}{A} - \frac{M_z}{I_z}\cdot y + \frac{M_y}{I_y}\, z}$$





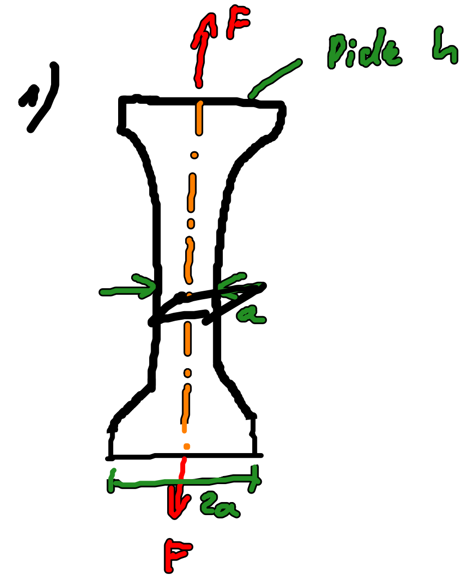
i) $N=0$: $\quad 0 = -\frac{M_z}{I_z}\cdot y + \frac{M_y}{I_y}\cdot z \implies z = \frac{M_z}{I_z}\frac{I_y}{M_y}\cdot y$



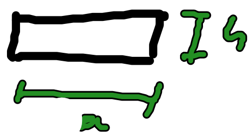
ii) $N \neq 0 \implies z = \frac{M_z}{I_z}\,\frac{I_y}{M_y}\cdot y - N\frac{I_y}{M_y}$

Beispiel 3:

1)

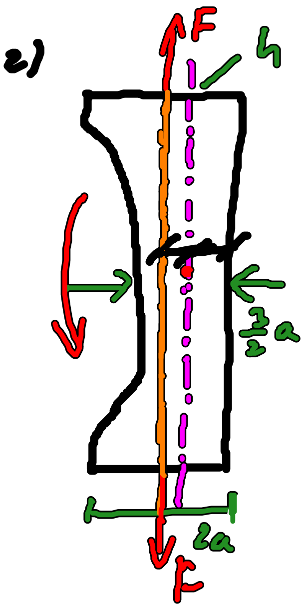


$$\sigma_1 = \frac{N}{A} = \frac{F}{A} = \frac{F}{ah}$$



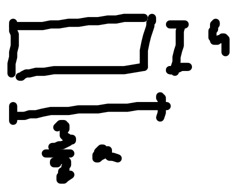
$$\sigma_2 = \frac{N}{A} + \frac{\overbrace{M_y}^{\text{Kraft wirkt im Sp}}}{I_y} \cdot z = \frac{F}{\frac{3}{2}ah} + \frac{F(a-\frac{3}{4}a)}{I_y} \cdot z$$

2)



$$= \frac{F}{\frac{3}{2}ah} + \frac{\frac{1}{4}Fa}{I_y} \cdot z$$

$$\sigma_{max} = \frac{F}{\frac{3}{2}ah} + \frac{Fa}{4\frac{1}{12}h(\frac{3}{2}a)^3} \cdot \frac{3}{4}a = \frac{4F}{3ah}$$



$$\sigma_{max}^1 = \frac{F}{ah}, \quad \sigma_{max}^2 = \frac{4F}{3ah} \Rightarrow \sigma_m^2 > \sigma_m^1$$