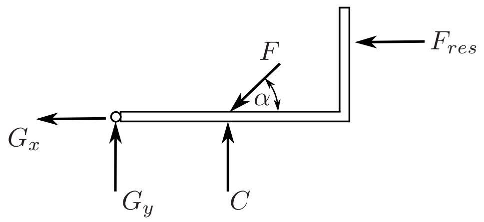
Aufgabe 1

(a)

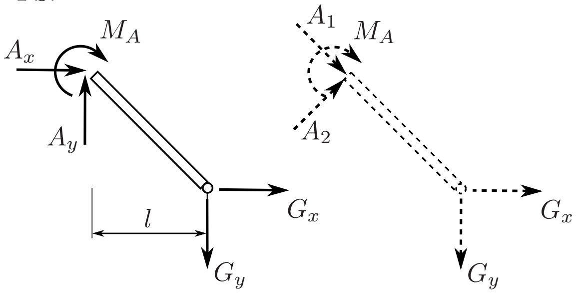
$$F_{res} = \frac{1}{2} q_0 l \tag{1}$$
$$y_{res} = \frac{2}{3} l \tag{2}$$

(b)
FS:



$$\sum M^B \stackrel{!}{=} 0, \quad 0 = Cl + \frac{1}{2} q_0 l \frac{2}{3} l - \sqrt{2} q_0 l \frac{\sqrt{2}}{2} \tag{3}$$
$$\Rightarrow C = \frac{2}{3} q_0 l \tag{4}$$
$$\sum F_y \stackrel{!}{=} 0, \quad 0 = \frac{2}{3} q_0 l + G_y - \sqrt{2} q_0 l \frac{\sqrt{2}}{2} \tag{5}$$
$$\Rightarrow G_y = \frac{1}{3} q_0 l \tag{6}$$
$$\sum F_x \stackrel{!}{=} 0, \quad 0 = -G_x - \sqrt{2} q_0 l \frac{\sqrt{2}}{2} - \frac{1}{2} q_0 l \tag{7}$$
$$\Rightarrow G_x = -\frac{3}{2} q_0 l \tag{8}$$

(c) FS:

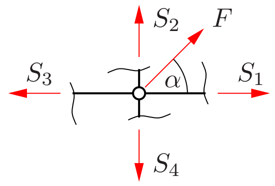


$$\sum M^A \stackrel{!}{=} 0, \quad 0 = M_A + \frac{1}{3} q_0 l^2 + \frac{3}{2} q_0 l^2 \tag{9}$$
$$\Rightarrow M_A = -\frac{11}{6} q_0 l^2 \tag{10}$$
$$\sum F_y \stackrel{!}{=} 0, \quad 0 = -\frac{1}{3} q_0 l + A_y \tag{11}$$
$$\Rightarrow A_y = \frac{1}{3} q_0 l \tag{12}$$
$$\sum F_x \stackrel{!}{=} 0, \quad 0 = A_x - \frac{3}{2} q_0 l \tag{13}$$
$$\Rightarrow A_x = \frac{3}{2} q_0 l \tag{14}$$

Alternativ können die Lagerkräfte $A_1 = \sqrt{2}\frac{7}{12} q_0 l$ und $A_2 = \sqrt{2}\frac{11}{12} q_0 l$ bestimmt werden.

Aufgabe 2

(a) FS und GGBen:



Materialstrukturge-

$$\sum F_x = 0 \tag{15}$$
$$S_1 - S_3 + F\cos\alpha = 0 \tag{16}$$
$$\sum F_y = 0 \tag{17}$$
$$S_2 - S_4 + F\sin\alpha = 0 \tag{18}$$

setz:

$$S_i = EA\frac{\Delta l_i}{l_i} \tag{19}$$

Querschnittsfläche und Längen der Stäbe:

$$A = D^2, \quad l_1 = d, \quad l_2 = b, \quad l_3 = c, \quad l_4 = a \tag{20}$$

Linearisierte Verformungskinematik:

$$\vec{u}_P = u_x \vec{e}_x + u_y \vec{e}_y \tag{21}$$

Die Längenänderungen der Stäbe sind:

$$\Delta l_1 = \vec{e}_1 \cdot \vec{u}_P = -\vec{e}_x \cdot \vec{u}_P = -u_x \tag{22}$$
$$\Delta l_2 = \vec{e}_2 \cdot \vec{u}_P = -\vec{e}_y \cdot \vec{u}_P = -u_y \tag{23}$$
$$\Delta l_3 = \vec{e}_3 \cdot \vec{u}_P = \vec{e}_x \cdot \vec{u}_P = u_x = -\Delta l_1 \tag{24}$$
$$\Delta l_4 = \vec{e}_4 \cdot \vec{u}_P = \vec{e}_y \cdot \vec{u}_P = u_y = -\Delta l_2 \tag{25}$$

Einsetzen in (19) liefert

$$S_1 = -\frac{EA}{d} u_x; \qquad S_3 = \frac{EA}{c} u_x; \tag{26}$$
$$S_2 = -\frac{EA}{b} u_y; \qquad S_4 = \frac{EA}{a} u_y; \tag{27}$$

Einsetzen der Gleichungen (26) in (16)

$$\Rightarrow \underline{\underline{u_x = \frac{F\cos\alpha}{EA(\frac{1}{c}+\frac{1}{d})}}} \tag{28}$$

Einsetzen der Gleichungen (27) in (18)

$$\Rightarrow \underline{\underline{u_y = \frac{F\sin\alpha}{EA(\frac{1}{a}+\frac{1}{b})}}} \tag{29}$$

(b) Es muß gelten $u_x = u_y$.

$$\frac{F\cos\alpha}{EA(\frac{1}{c}+\frac{1}{d})} = \frac{F\sin\alpha}{EA(\frac{1}{a}+\frac{1}{b})} \tag{30}$$
$$\Rightarrow \alpha = \arctan\frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{c}+\frac{1}{d}} \tag{31}$$

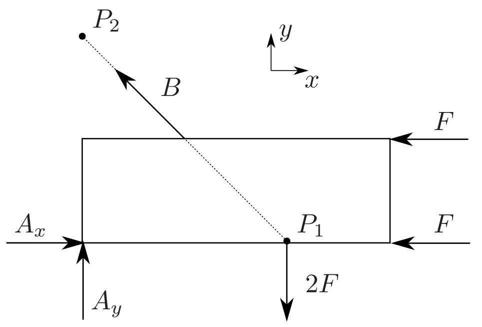
Aufgabe 3

(a)

- $2k = r + s \Rightarrow 2 \cdot 8 = 3 + 13$ ✓
- nicht verschieblich oder vorspannbar

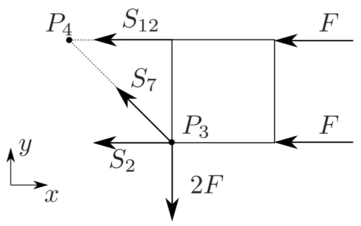
(b) $4, 11, \quad 6, 10 \quad 9$

(c)



$$\sum M^{(P_1)} = 0 = -A_y 2a + Fa \Rightarrow A_y = \frac{1}{2}F$$
$$\sum M^{(A)} = 0 = -2F2a + Fa + \frac{\sqrt{2}}{2}B \cdot 2a \Rightarrow B = \frac{3}{\sqrt{2}}F$$
$$\sum M^{(P_2)} = 0 = A_x 2a - 2F2a - F2a - Fa \Rightarrow A_x = \frac{7}{2}F$$

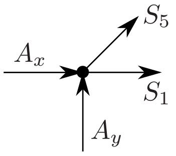
(d)



$$\sum M^{(P_3)} = 0 = S_{12}a + Fa \qquad \Rightarrow S_{12} = -F$$
$$\sum M^{(P_4)} = 0 = -S_2 a - Fa - 2Fa \qquad \Rightarrow S_2 = -3F$$
$$\sum F_y = 0 = S_7 \frac{\sqrt{2}}{2} - 2F \qquad \Rightarrow S_7 = 2\sqrt{2}F$$

(e) durch hinsehen:

$$S_1 = S_2 = -3F$$
$$S_3 = -F$$
$$S_{13} = -3F$$
$$S_8 = -2F$$



$$\Rightarrow S_5 = -\frac{2}{\sqrt{2}}A_y = -\frac{1}{\sqrt{2}}F$$

Die Stäbe aus (e) sind Druckstäbe.

Aufgabe 4

(a) Biegeliniendifferentialgleichung:

$$EIw''''(x) = q(x), \tag{32}$$

mit Streckenlast dieser Aufgabe:

$$EIw''''(x) = q_0 \frac{x}{l}. \tag{33}$$

(b) Integrieren von Gleichung (33):

$$EIw'''(x) = \frac{1}{2}\frac{q_0}{l}x^2 + C_1 \tag{34}$$
$$EIw''(x) = \frac{1}{6}\frac{q_0}{l}x^3 + C_1 x + C_2 \tag{35}$$
$$EIw'(x) = \frac{1}{24}\frac{q_0}{l}x^4 + \frac{1}{2}C_1 x^2 + C_2 x + C_3 \tag{36}$$
$$EIw(x) = \frac{1}{120}\frac{q_0}{l}x^5 + \frac{1}{6}C_1 x^3 + \frac{1}{2}C_2 x^2 + C_3 x + C_4 \tag{37}$$

Dynamische Randbedingungen:

$$-EIw'''(x=0) = F \tag{38}$$
$$EIw''(x=0) = 0 \tag{39}$$

Geometrische Randbedingungen:

$$w(x=l) = 0 \tag{40}$$
$$w'(x=l) = 0 \tag{41}$$

(c) Aus (38) und (39):

$$C_1 = -F \quad \text{und:} \quad C_2 = 0 \tag{42}$$

(40) in (36) und (41) in (37):

$$C_3 = -\frac{1}{24}q_0 l^3 + \frac{1}{2}Fl^2, \quad C_4 = \frac{1}{30}q_0 l^4 - \frac{1}{3}Fl^3 \tag{43}$$

Aus (36) mit $\varphi_A = w'(x=0)$:

$$\Rightarrow \underline{\underline{\varphi_A = \left(-\frac{1}{24}q_0 l^3 + \frac{1}{2}Fl^2\right)\frac{1}{EI}}} \tag{44}$$

(d) Gefordert ist:

$$w(0) \stackrel{!}{=} 0 \tag{45}$$

(43) in (37) eingesetzt und nach F aufgelöst:

$$\Rightarrow \underline{\underline{F = \frac{1}{10}q_0 l}} \tag{46}$$

Aufgabe 5

(a) Oberes Dreieck als 1, unteres als 2 nummeriert.

i	z_i	A_i	$z_i A_i$
1	$\frac{2}{3}a$	a^2	$\frac{2}{3}a^3$
2	$2a$	$3a^2$	$6a^3$
$\sum$	–	$4a^2$	$\frac{20}{3}a^3$

$$\Rightarrow \tilde{z}_S = \frac{\frac{20}{3}a^3}{4a^2} = \frac{5}{3}a$$

(b)

i	z_{Si}	$z_{Si}^2 A_i$	$I_{\tilde{y}\tilde{y}i}$
1	a	a^4	$\frac{1}{18}a^4$
2	$\frac{1}{3}a$	$\frac{1}{3}a^4$	$\frac{27}{18}a^4$
$\sum$	–	$\frac{4}{3}a^4$	$\frac{28}{18}a^4$

$$\Rightarrow I_{yy} = \frac{4}{3}a^4 + \frac{28}{18}a^4 = \frac{26}{9}a^4$$

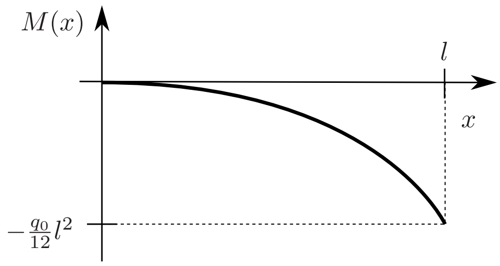
(c)

$$Q'(x) = -q(x)$$

$$Q = -\int \frac{q_0}{l^2}x^2 \mathrm{d}x = -\frac{q_0}{3l^2}x^3 + C_1$$
$$M = \int Q \mathrm{d}x = -\frac{q_0}{12l^2}x^4 + C_1 x + C_2$$

RB: $M(0) = 0 \;\Rightarrow\; C_2 = 0$
$M(l) = -M_0 \;\Rightarrow\; C_1 = 0$

$$\Rightarrow M(x) = -\frac{q_0}{12l^2}x^4$$



(d)

$$\sigma(x,z) = \frac{N}{A} + \frac{M_y}{I_y}z$$

$$x = l\;:\quad \sigma(z) = \frac{-F}{A} - \frac{q_0 l^2}{12 I_y}z$$

Druckspannung für alle $z \Rightarrow$ kritisch ist Oberkante:

$$\sigma(z=-2a) = 0$$
$$0 = -\frac{F}{4a^2} - \frac{q_0 l^2}{12\cdot 3a^4}\cdot(-2a)$$
$$F = \frac{2}{9}\frac{q_0 l^2}{a}$$

Alternativ mit den Werten aus (a) und (b):

$$F = \frac{5}{26}\frac{q_0 l^2}{a}$$

Aufgabe 6

(a)
DGL:

$$w'''' = -\lambda^2 w'' \quad \text{mit} \quad \lambda^2 = \frac{F}{EI} \tag{47}$$

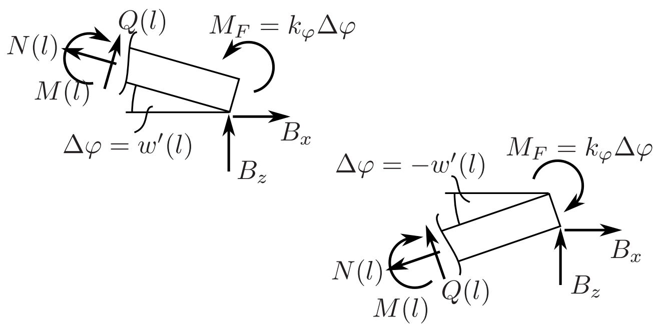
Lösung:

$$w(x) = A\cos\lambda x + B\sin\lambda x + Cx + D \tag{48}$$

(b)
Randbedingungen:

$$\left.\begin{array}{r} w(0) = 0 \\ w(l) = l \end{array}\right\} \tag{49}$$
$$M(0) = -EIw''(0) = 0 \tag{50}$$

FS bei $x = 0$ pos. ($w' > 0$) oder neg. ($w' < 0$) ausgelenkt.



$\Delta\varphi = w'(l)$ oder $\Delta\varphi = -w'(l)$

Damit ergibt sich aus dem Momentengleichgewicht:

$$-EIw''(l) = k_\varphi w'(l) \tag{51}$$

Ableitungen:

$$w'(x) = -\lambda A\sin\lambda x + \lambda B\cos\lambda x + C \tag{52}$$
$$w''(x) = -\lambda^2 A\cos\lambda x - \lambda^2 B\sin\lambda x \tag{53}$$

Aus (49a) und (50) folgt:

$$0 = A + D \tag{54}$$
$$0 = A \quad\Rightarrow\quad D = 0 \tag{55}$$

Aus (49b) folgt:

$$0 = B\sin\lambda l + Cl \quad\Rightarrow\quad C = -\frac{B}{l}\sin\lambda l \tag{56}$$

Aus (51) folgt:

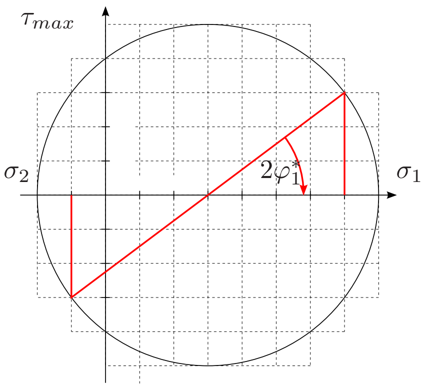
$$EI\lambda^2 B\sin\lambda l = k_\varphi\lambda B\cos\lambda l - k_\varphi\frac{B}{l}\sin\lambda l \tag{57}$$

$$\Rightarrow \quad \sin\lambda l\left(EI\lambda^2 + \frac{k_\varphi}{l}\right) = k_\varphi\lambda\cos\lambda l \tag{58}$$

$$\Leftrightarrow \tan\lambda l = \frac{k_\varphi\lambda}{EI\lambda^2 + \frac{k_\varphi}{l}} \tag{59}$$

$$= \frac{k_\varphi\lambda l}{\frac{EI}{l}(\lambda l)^2 + k_\varphi} \tag{60}$$

Aufgabe 7



(a)

$$\underline{\sigma} = \begin{pmatrix} 7\sigma_0 & 3\sigma_0 \\ 3\sigma_0 & -\sigma_0 \end{pmatrix}_{(x,y)}$$

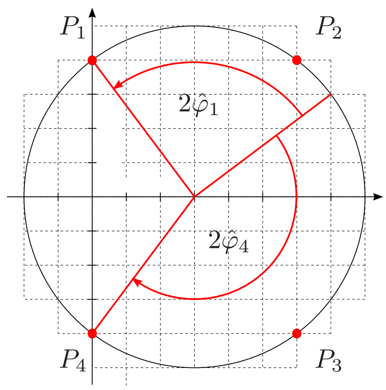
(b)

$$\sigma_1 = 8\sigma_0 \qquad \sigma_2 = -2\sigma_0$$
$$\tau_{max} = 5\sigma_0$$

(c)

$$tan(2\varphi_1^*) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{3}{4}$$
$$\Rightarrow \varphi_1^* = 17,5^\circ$$
$$\varphi_2^* = \varphi_1^* + 90^\circ = 107,5^\circ$$

(d)



$$2\hat{\varphi}_1 = -90^\circ \Rightarrow \hat{\varphi}_1 = \underline{-45^\circ}, \ (135^\circ)$$
$$2\hat{\varphi}_2 = -16^\circ \Rightarrow \hat{\varphi}_2 = \underline{-8^\circ}, \ (172^\circ)$$
$$2\hat{\varphi}_3 = 2\hat{\varphi}_1 + 180^\circ \Rightarrow \hat{\varphi}_3 = \underline{45^\circ}$$
$$2\hat{\varphi}_4 = 2(2\varphi_1^*) + 90^\circ = 164^\circ \Rightarrow \hat{\varphi}_4 = \underline{82^\circ}$$

$$P_1: \quad \hat{\varphi} = \hat{\varphi}_1 = -45^\circ, (135^\circ) \quad \Rightarrow \quad \underline{\sigma} = \begin{pmatrix} 0 & +4\sigma_0 \\ +4\sigma_0 & +6\sigma_0 \end{pmatrix}_{(\xi,\eta)}$$

$$P_2: \quad \hat{\varphi} = \hat{\varphi}_2 = -8^\circ, \ (172^\circ) \quad \Rightarrow \quad \underline{\sigma} = \begin{pmatrix} +6\sigma_0 & +4\sigma_0 \\ +4\sigma_0 & 0 \end{pmatrix}_{(\xi,\eta)}$$

$$P_3: \quad \hat{\varphi} = \hat{\varphi}_3 = 45^\circ \quad \Rightarrow \quad \underline{\sigma} = \begin{pmatrix} +6\sigma_0 & -4\sigma_0 \\ -4\sigma_0 & 0 \end{pmatrix}_{(\xi,\eta)}$$

$$P_4: \quad \hat{\varphi} = \hat{\varphi}_4 = 82^\circ \quad \Rightarrow \quad \underline{\sigma} = \begin{pmatrix} 0 & -4\sigma_0 \\ -4\sigma_0 & +6\sigma_0 \end{pmatrix}_{(\xi,\eta)}$$