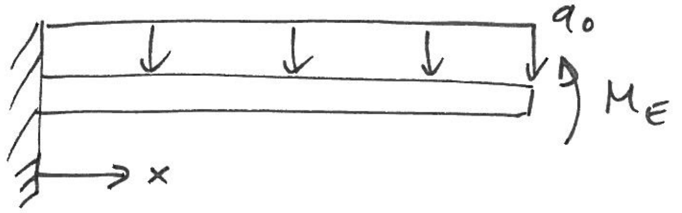
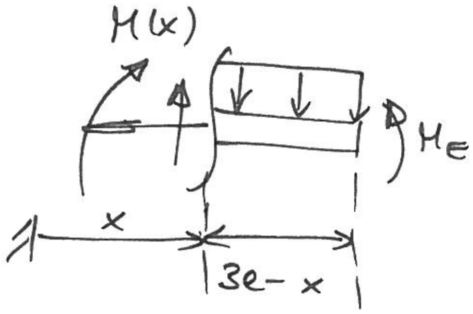
AUFGABE 69:

I) BESTIMMEN DES LAGERMOMENTS

1) Bestimmen des Biegemoments

mit $\frac{\partial W}{\partial M_E} \stackrel{!}{=} 0$





$$M(x) = -\frac{q_0}{2}(3l-x)^2 + M_E$$

2) Bestimmung $\frac{\partial W}{\partial M_E}$

$$W = \frac{1}{2}\int_0^{3l} \frac{M(x)^2}{EI}\,dx$$

$$\frac{\partial W}{\partial M_E} = \frac{1}{EI}\int_0^{3l} M(x)\frac{\partial M(x)}{\partial M_E}\,dx \stackrel{!}{=} 0$$

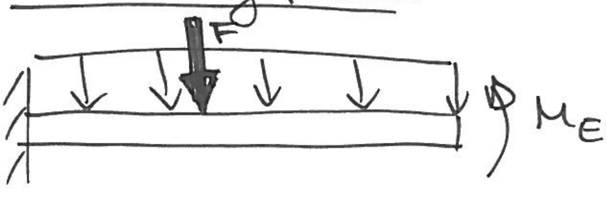
$$= \frac{1}{EI}\int_0^{3l}\left(M_E - \frac{q_0}{2}(3l-x)^2\right)dx$$

$$= \frac{1}{EI}\left[M_E\,\cancel{3l} - \frac{q_0\,(3l)^{\cancel{3}\,2}}{6}\right]$$

$$\Rightarrow \boxed{M_E = \frac{3q_0 l^2}{2}}$$

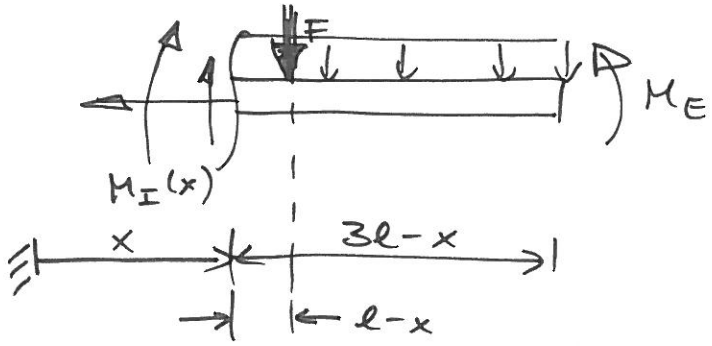
II) BESTIMMEN DER DURCHSENKUNG

1) Ersatzsystem



$$\frac{\partial W}{\partial F} = w(x = l)$$

② Bestimmen des Biegemoments



$$M_I(x) = M_E - F(l-x) - \frac{q_0}{2}(3l-x)^2$$

$$M_{II}(x) = M_E - \frac{q_0}{2}(3l-x)^2$$

③ Castigliano anwenden:

$$\frac{\partial W}{\partial F} = \frac{1}{EI}\int_0^l M_I \frac{\partial M_I}{\partial F}\,dx + \frac{1}{EI}\int_l^{3l} M_{II}\frac{\partial M_{II}}{\partial F}\,dx$$

mit $\frac{\partial M_I}{\partial F} = (x-l)$ und $\frac{\partial M_{II}}{\partial F} = 0$

folgt

$$\left.\frac{\partial W}{\partial F}\right|_{F=0} = \frac{1}{EI}\int_0^l \left[\frac{3}{2}q_0 l^2 (x-l) - \frac{q_0}{2}(3l-x)^2(x-l)\right]dx$$

$$= \frac{1}{EI}\int_0^l (x-l)\left[\underbrace{\frac{3}{2}q_0 l^2 - \frac{9}{2}q_0 l^2}_{-3q_0 l^2} + 3q_0 l x - \frac{q_0 x^2}{2}\right]$$

$$= \frac{1}{EI}\int_0^l \left[-6 q_0 l^2 x + 3 q_0 l^3 + \frac{7}{2} q_0 l x^2 - \frac{q_0 x^3}{2}\right]dx$$

$$= \frac{1}{EI}\left[-3q_0 l^4 + 3q_0 l^4 + \frac{7}{6}q_0 l^4 - \frac{q_0 l^4}{8}\right]$$

$$= \frac{1}{EI}\left[\frac{28}{24} - \frac{3}{24}\right]q_0 l^4$$

$$\boxed{w(x=l) = \frac{25}{24} q_0 l^4}$$