Tutorium: 113, 115

HA: 114, 118

Flächenträgheitsmomente

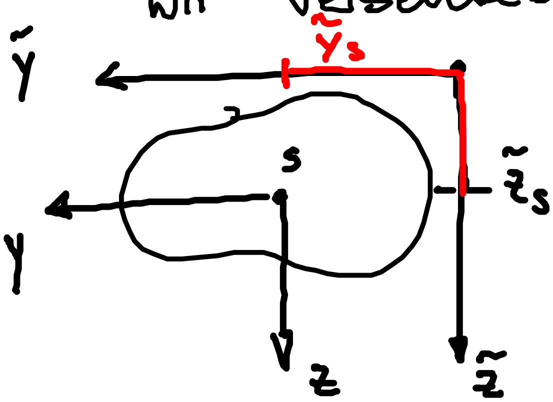
$$I_y = \int z^2 \, dA$$

$$I_z = \int y^2 \, dA$$

$$I_{yz} = -\int yz \, dA$$

$$\begin{pmatrix} I_y & I_{yz} \\ I_{yz} & I_z \end{pmatrix} \rightarrow \begin{pmatrix} I_1 & 0 \\ 0 & I_2 \end{pmatrix}$$

wir verschieben die Achsen parallel

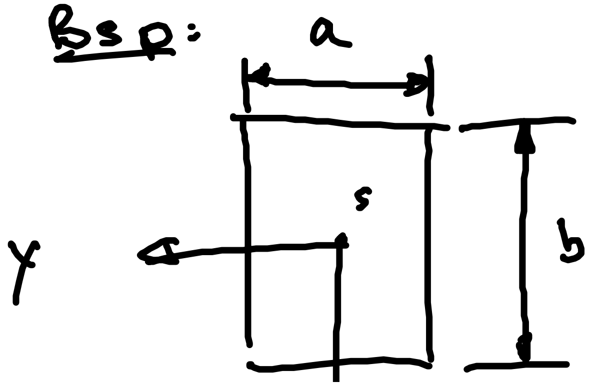


Satz von Steiner:

$$\boxed{\begin{aligned} I_{\tilde{y}} &= I_y + \tilde{z}_s^2 A \\ I_{\tilde{z}} &= I_z + \tilde{y}_s^2 A \\ I_{\tilde{y}\tilde{z}} &= I_{yz} - \tilde{y}_s \tilde{z}_s A \end{aligned}}$$

Steiner Anteil

Bsp:



$$I_y = \frac{1}{12} a b^3$$

$$I_z = \frac{1}{12} b a^3$$

$$I_{yz} = 0$$

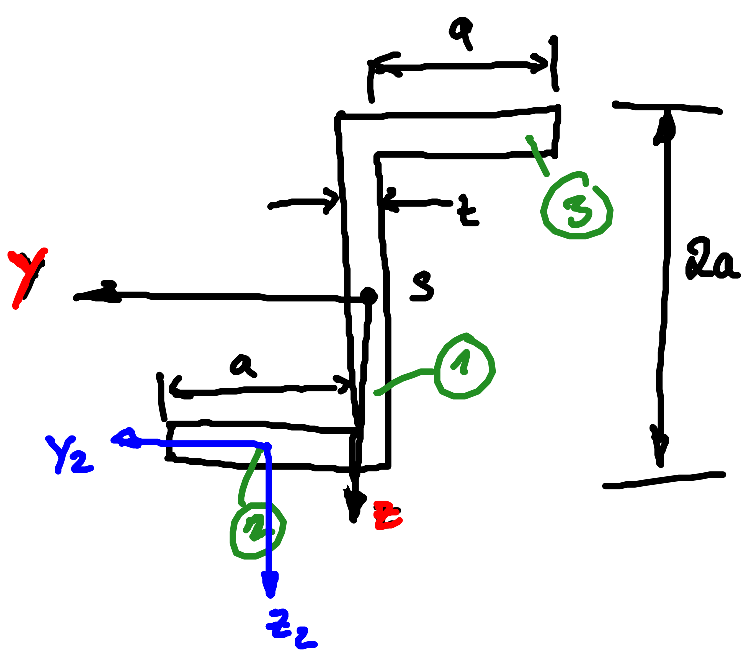
$$M_y(x) = -EI_y\, w''(x)$$

$$M_z(x) = -EI_z\, v''(x)$$

$$M_y = M_z \quad \Rightarrow \quad v > w$$

weil $I_y > I_z$

Bsp: siehe VL



$t \ll a$

↳ Dünnwandig

① Ermitteln des Schwerpunktsystems

② Verwende erweitertes Tabellenverfahren

③ I_y, I_z, I_{yz}

① fällt weg

② Tabellenverfahren

$$I_y = I_{y1} + I_{y2} + I_{y3}$$

I_y

	A_i	z_{si}	I_{zi}	$z_{si}^2 \, A_i$
1	$2at$	0	$\frac{1}{12} t (2a)^3$	0
2	at	$(a - \frac{t}{2})$	$\frac{1}{12} a \cdot t^3$	$(a - \frac{t}{2})^2 \cdot at$
3	at	$-(a - \frac{t}{2})$	$\frac{1}{12} a \cdot t^3$	$(a - \frac{t}{2})^2 \, at$
Σ	/	/	$\frac{1}{6} a \cdot t^3 + \frac{1}{12} 8a^3 t$	$2(a - \frac{t}{2})^2 at$

$$I_y = I_{y1} + I_{y2} + z_{s2}^2 A_2 + I_{y3} + z_{s3} A_3$$

$$I_y = \frac{1}{6} a \cdot \cancel{t^3}^{\approx 0} + \frac{2}{3} a^3 t + 2(a^2 - ta + \underbrace{\frac{t^2}{4}}_{\approx 0}) at$$

≈ 0

$t << a \qquad t^2 << 0 \qquad t^3 <<< 0$

$$I_y = \frac{2}{3} a^3 t + 2a^3 t$$

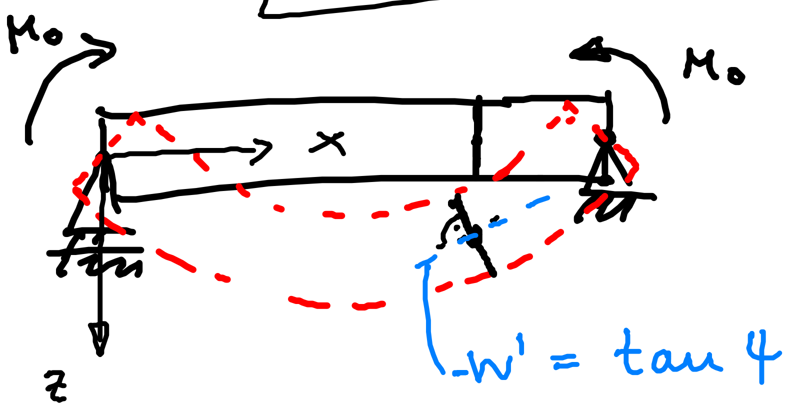
$$\boxed{I_y = \frac{8}{3} a^3 t}$$

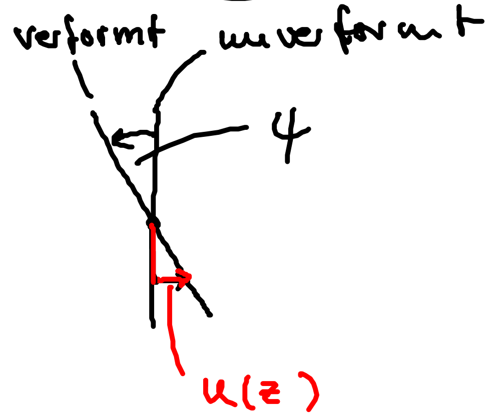
I_z, I_{yz} analog

Biegespannung

$$-EI_y w''(x) = +M_y(x) \longleftrightarrow \boxed{\sigma = E\,\varepsilon}$$

Biegespannung





kleine w $\quad -w' \approx \varphi$ $\qquad \varepsilon = \dfrac{du}{dx}$

$$u(z) = z \cdot \varphi$$

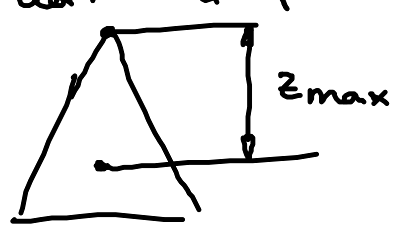
$$u(z) = -z\,w' \qquad \varepsilon = -z\,w''.$$

$$\sigma = -E z w'' \quad \Rightarrow \quad w'' = -\frac{\sigma}{E z}$$

$$M_y(x) = \cancel{E} I_y \frac{\sigma}{\cancel{E} z}$$

$$\boxed{\sigma(x) = \frac{M_y(x)}{I_y(x)}\, z}$$

Die maximalen Spannungen treten dort auf wo z am größten ist



$$\underline{\sigma_{max}} = \frac{M_y(x)}{W} \qquad W := \frac{I_y}{z_{max}}$$

Vergleich zwischen Belastungsarten

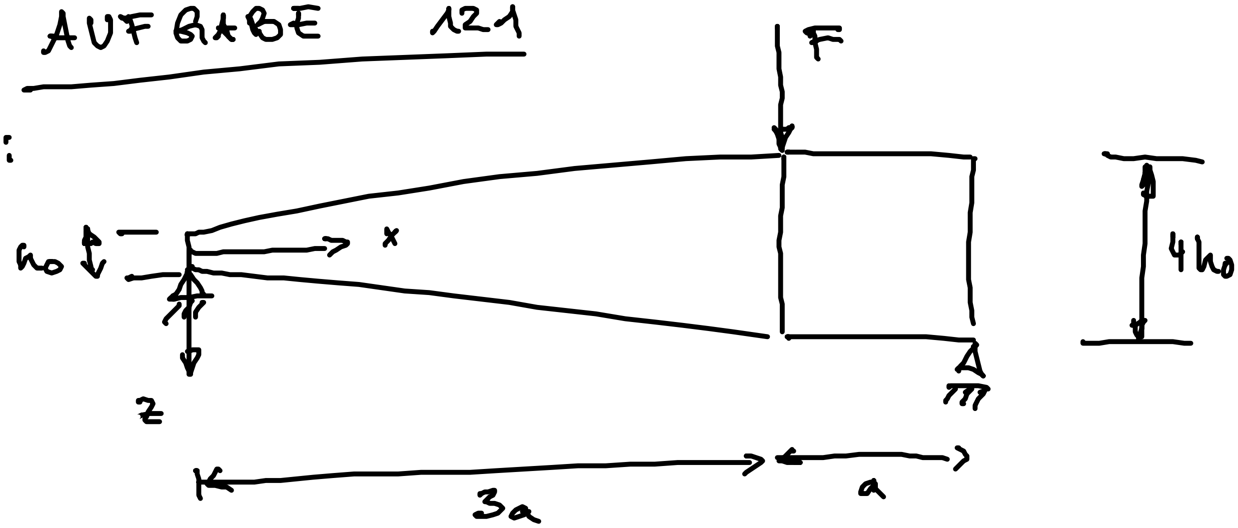
	Dehnung	Torsion	Biegung
Verschiebung	u	ϑ	w
Dehnungen Gleitungen	$\varepsilon = \frac{du}{dx}$	$\gamma = r \frac{d\vartheta}{dx}$	$\varepsilon = -z \frac{dw}{dx}$
Materialgesetz (HOOKE)	$\sigma = E \varepsilon$	$\tau = G \gamma$	$\sigma = E \varepsilon$
innere Lasten	$N = EA\, u'$	$M_t = GI_p\, \vartheta'$	$M_y(x) = -EI_y\, w''(x)$

QUIZ

③ Tritt die maximale Spannung immer dort auf, wo $M_y(x)$ maximal ist?

AUFGABE 121

Geg:



Ges: $\sigma(x)$

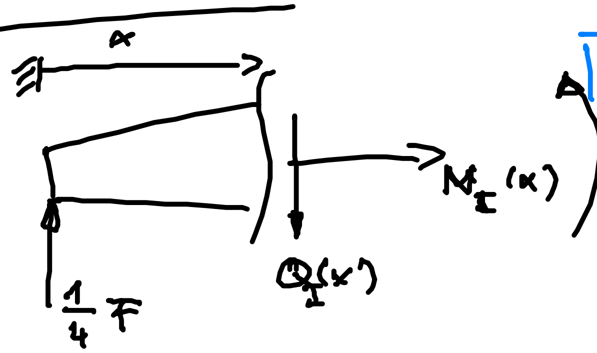
insbesondere $|\sigma_{max}|$

$$\sigma(x) = \frac{M_y(x)}{I_y(x)} z$$

1. $M_y(x)$ bestimmen
2. $I_y(x)$ bestimmen
3. $\sigma(x)$ — " —
4. σ_{max} bei maximalen z

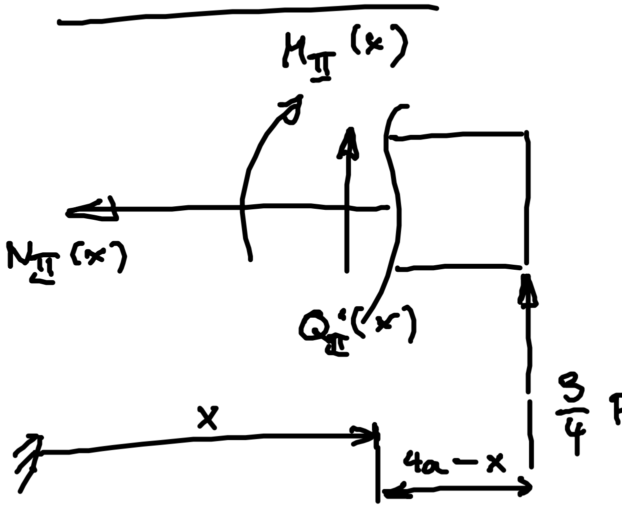
① $M_y(x)$:

Bereich I: $0 \leq x \leq 3a$



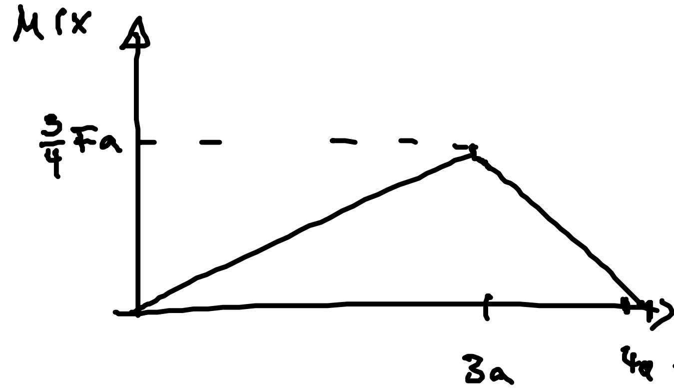
$$\boxed{M_I(x) = \frac{1}{4} F x}$$

Bereich II:



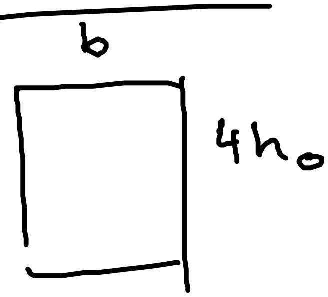
$$M_{II}(x) = \frac{3}{4}F(4a - x)$$

$$\boxed{M_{II}(x) = -\frac{3}{4}Fx + 3aF}$$



② I_y

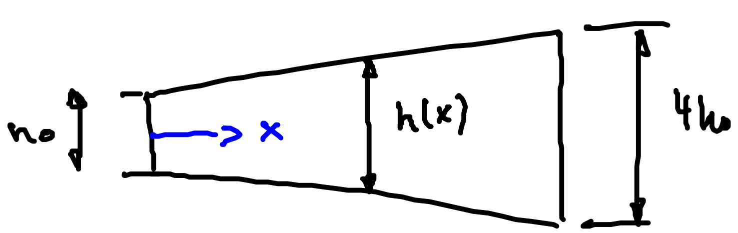
Bereich II



$$I_{y,II} = \frac{1}{12} b (4h_0)^3$$

$$\boxed{I_{y,II} = \frac{16}{3} b h_0^3}$$

Bereich I:



$$I_{y,I}(x) = \frac{1}{12} b\, h(x)^3$$

$$\frac{h(x)}{2} = \frac{h_0}{2} + \frac{2h_0 - \frac{h_0}{2}}{3a} x$$

$$\frac{h(x)}{2} = \frac{h_0}{2} + \frac{1}{2}\frac{h_0}{a}x \quad \Big|\cdot 2$$

$$\boxed{h(x) = h_0 + \frac{h_0}{a}x}$$

$$I_{y,I}(x) = \frac{1}{12} b \left(\frac{h_0}{a}x + h_0\right)^3$$

③ $\underline{\sigma(x)}$

$$\sigma_I(x) = \frac{\frac{1}{4}Fx}{\frac{1}{12} b \left(\frac{h_0}{a}x + h_0\right)^3} \cdot z$$

$$\sigma_I(x) = \frac{3Fx}{b\left(\frac{h_0}{a}x + h_0\right)^3} \cdot z$$

$$\sigma_{I,max}\left(x, z = \frac{h(x)}{2}\right) = \frac{3Fx}{2b\left(\frac{h_0}{a}x + h_0\right)^2}$$

$$\sigma_{II}(x) = \frac{-\frac{3}{4}Fx + 3Fa}{\frac{16}{3} b h_0^3} z$$

$$\sigma_{II,max}(x, z = 2h_0) = \frac{-\frac{3}{4}Fx + 3Fa}{\frac{16}{3} b h_0^2} 2$$

④ Maximale Spannung:

Bereich II:

$$\sigma_{II max}(x = 3a, z = 2h_0) = 2\frac{\frac{3}{4}Fa}{\frac{16}{3} b h_0^3} \cdot \cancel{2}h_0$$

$$= \frac{9}{32} \frac{F a}{b h_0^2}$$

Bereich I:

$$\frac{d \sigma_I(x)}{dx} \overset{!}{=} 0$$

$$\frac{d \sigma_I(x)}{dx} = \frac{3}{2} \frac{F}{b} \left[\frac{\overbrace{1 \cdot \left(\frac{h_0}{a} x + h_0\right)^2 - x \frac{h_0}{a} 2 \left(\frac{h_0}{a} x + h_0\right)}^{= 0}}{\left(\frac{h_0}{a} x + h_0\right)^4} \right]$$

$$\left(\frac{h_0}{a} x_E + h_0\right)^2 - x_E \frac{h_0}{a} 2 \left(\frac{h_0}{a} x_E + h_0\right) = 0$$

$$\underbrace{\left(\frac{h_0}{a} x_E + h_0\right)}_{= 0} \underbrace{\left[\left(\frac{h_0}{a} x_E + h_0\right) - 2 \frac{h_0}{a} x_E\right]}_{= 0} = 0$$

$$x_{E,1} = -a \qquad \boxed{x_{E,2} = +a}$$

$$\sigma_{I\,max}\left(x = a,\ z = \frac{h(a)}{2}\right) = \frac{3}{8} \frac{F a}{b h_0^2}$$

$$\sigma_I(x = 0) = 0$$

$$\sigma_I(x = 3a) = \sigma_{II,max}(x = 3a)$$

$$\boxed{\sigma_{I,max}\left(x = a,\ z = \frac{h(a)}{2}\right)} > \sigma_{II,max}(x = 3a)$$