3. Plenarübung: Schwerpunkt

AUFGABEN:

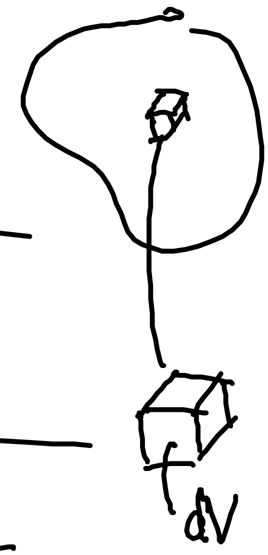
ÜE: 21, 25

TUT: 22a, 27

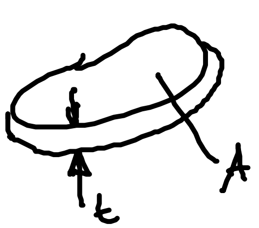
HA: 22b, 26, 28

Einführung Schwerpunkt

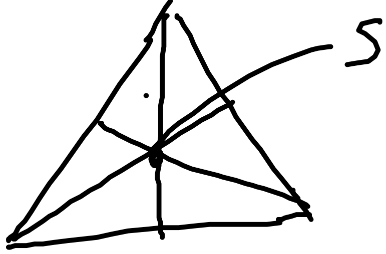
	diskret	kontinuierlich
allgemein	$x_s = \dfrac{\sum x_i m_i}{\sum m_i}$	$x_s = \dfrac{\int_m x \, dm}{\int dm}$
$m = \rho V$	$x_s = \dfrac{\sum x_i \rho_i V_i}{\sum \rho_i V_i}$	$x_s = \dfrac{\int_m x \, d(\rho V)}{\int d(\rho V)}$
$\rho = const$ homogenes Material	$x_s = \dfrac{\sum x_i V_i}{\sum V_i}$	$x_s = \dfrac{\int x dV}{\int dV}$
Ebenes Problem $V = A t$	$x_s = \dfrac{\sum x_i A_i}{\sum A_i}$	$x_s = \dfrac{\int x dA}{\int dA}$



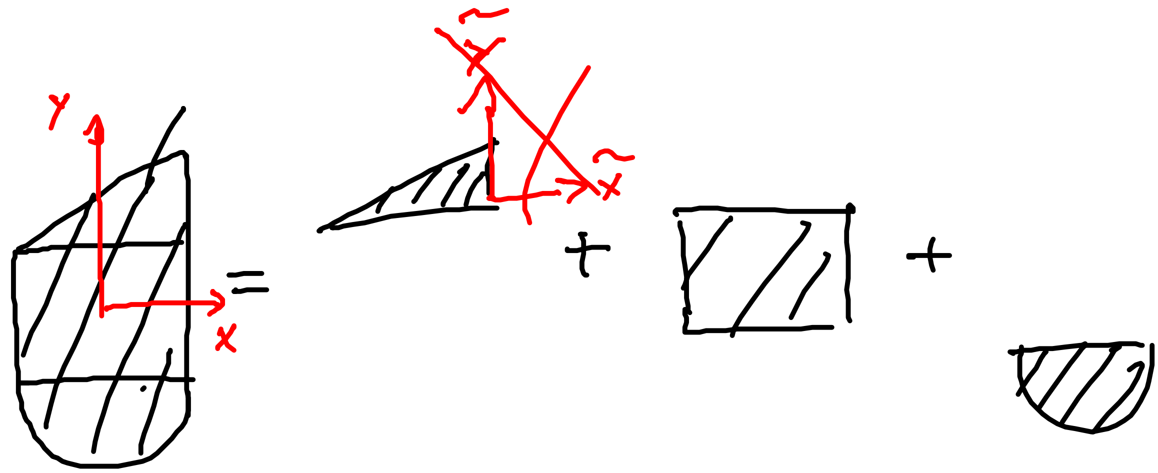
Flächenmittelpunkt



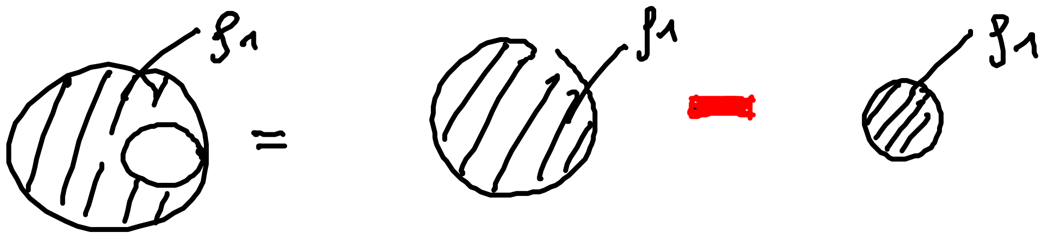
y_s, z_s – Komponenten sind analog definiert.



Bsp 1:



Bsp 2



Rechenweg

1) Körper in bekannte Teilkörper zerlegen

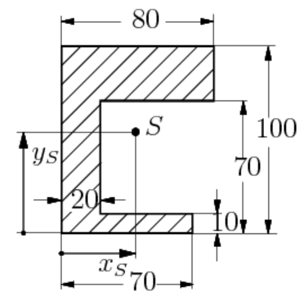
2) x_i, $x_i A_i$, y_i, $y_i A_i$ ... bestimmen (TABELLE)

3) Summen bilden

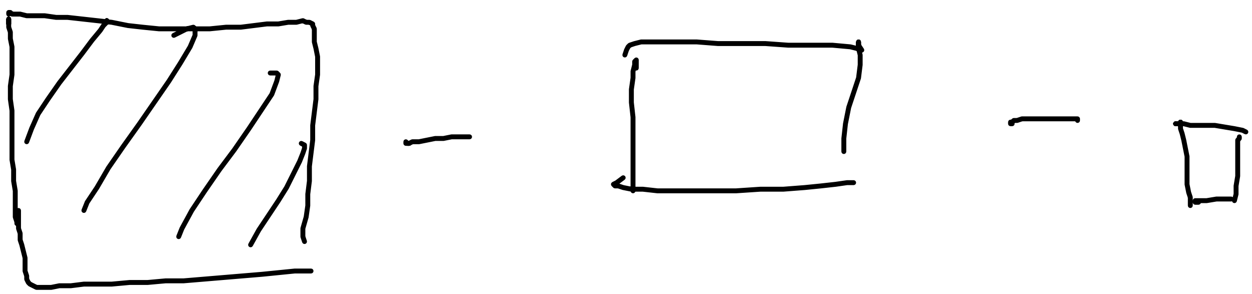
4) Schwerpunkts Koordinate bestimmen

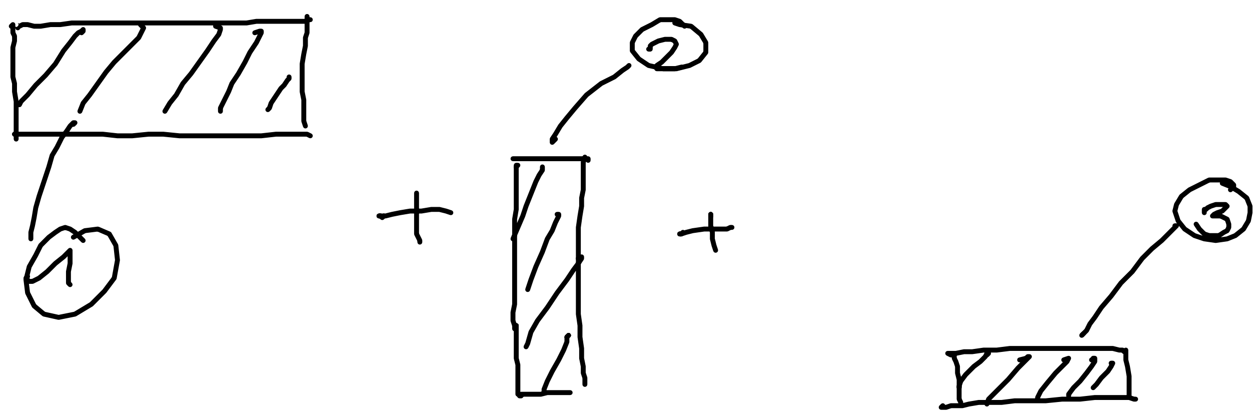
AUFGABE 21

1. Es sind die Schwerpunktabstände x_S und y_S des nebenstehend skizzierten Blechteiles zu bestimmen. (Dicke $d = 3$mm)



1) Körper unterteilen:





2) Tabelle aufstellen

$$x_S = \frac{\sum x_i A_i}{\sum A_i} \qquad y_S = \frac{\sum y_i A_i}{\sum A_i}$$

	x_i	y_i	A_i	$x_i A_i$	$y_i A_i$
1	4	8,5 cm	24	96	204
2	1	3,5	14	14	49
3	4,5	0,5	5	22,5	2,5
3) $\sum$	X	X	43 cm^2	132,5 cm^3	255,5 cm^3

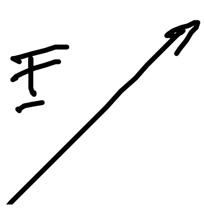
4) $$x_S = \frac{\sum x_i A_i}{\sum A_i} = \frac{132{,}5\ cm^{\cancel{3}}}{43\ \cancel{cm^2}} = \underline{\underline{3{,}08\ cm}}$$

$$y_S = \frac{\sum y_i A_i}{\sum A_i} = \frac{255{,}5\ cm^3}{43\ cm^2} = 5{,}94\ cm$$

QUIZ:

$$\underline{F} = F\underline{e}_x - 2F\underline{e}_z$$

$\underline{F}$



$$\underline{M}^{(P)} = ? \quad \underline{0}$$

$P(a, -a, 0)$ $\quad \underline{M}^{(0)} = \underline{r} \times \underline{F}$

---

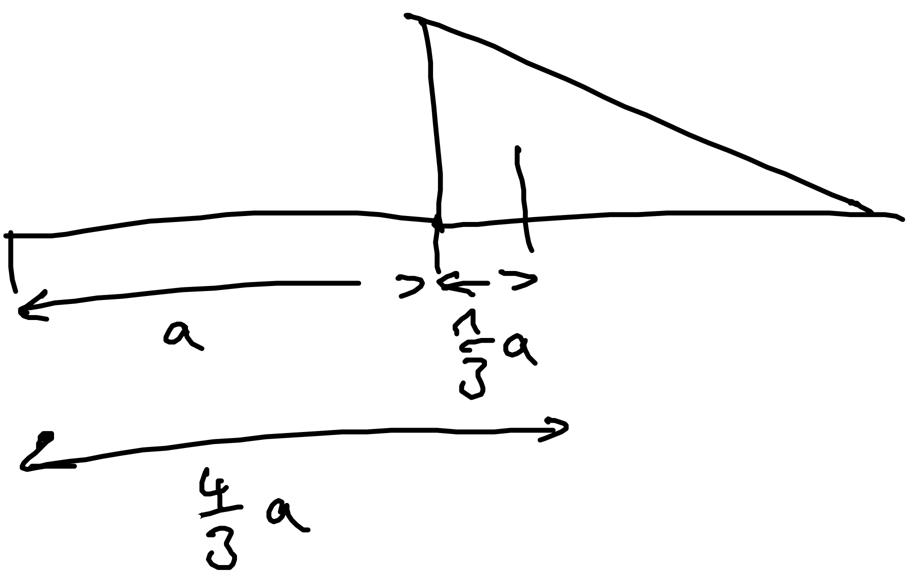
2) $\quad x_S = \frac{17}{5} r = \frac{\sum A_i x_i}{\sum A}$

$$= \frac{\pi r^2 r + \pi (2r)^2 \cdot 4r}{\pi r^2 + (2r)^2 \pi}$$

$$y_S = 0$$

---

3) $\quad x_S = \frac{4}{3} a$

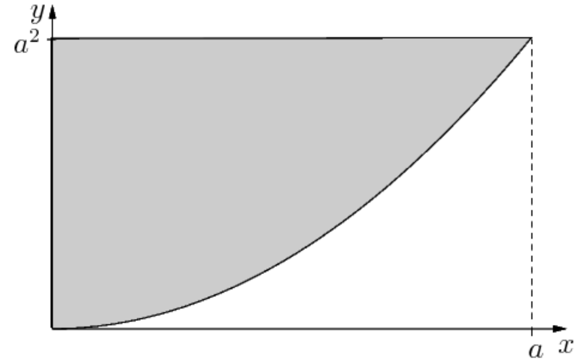


AUFGABE 25 (modifiziert)

2. Berechnen Sie die Koordinaten des Mittelpunkts der Fläche, die durch den Graphen der Normalparabel, die y-Achse und die Linie $\cancel{y =}\, a^2$ begrenzt wird (s. Skizze).

   (a) Stellen Sie die Funktionsgleichung der Normalparabel auf.

   (b) Berechnen Sie alle notwendigen Integrale.



Geg.: a

a) Funktionsgleichung aufstellen

$$y(x) = Ax^2 + Bx + C$$

$$y(0) = 0 \Rightarrow 0 + 0 + \boxed{C = 0}$$

$$y(a) = a \Rightarrow A \cdot a^2 + Ba = a \quad (1)$$

$$y(-a) = a \Rightarrow A \cdot a^2 - Ba = a \quad (2)$$

$$(1) - (2) \Rightarrow 2Ba = 0$$

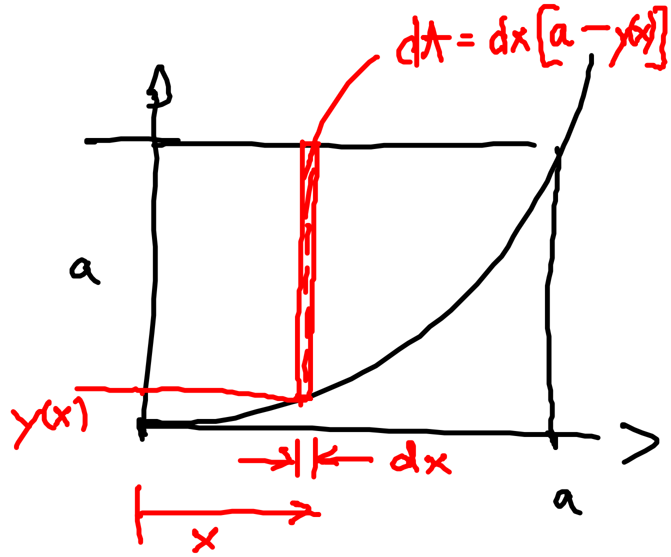
$$\boxed{B = 0}$$

$$(1) + (2) \Rightarrow 2A \cdot a^2 = 2a$$

$$\boxed{A = \frac{1}{a}}$$

$$y(x) = \frac{1}{a} x^2$$

b) Schwerpunktskoordinaten bestimmen



$$\underline{x_s = \frac{\int x\, dA}{\int dA}}$$

$$x_s = \frac{\int_0^a x\,[a - y(x)]\, dx}{\int_0^a [a - y(x)]\, dx}$$

$$= \frac{\int_0^a x\left[a - \frac{1}{a}x^2\right] dx}{\int_0^a \left[a - \frac{1}{a}x^2\right] dx}$$

$$= \frac{\int_0^a \left[ax - \frac{1}{a}x^3\right] dx}{\int_0^a \left[a - \frac{1}{a}x^2\right] dx}$$

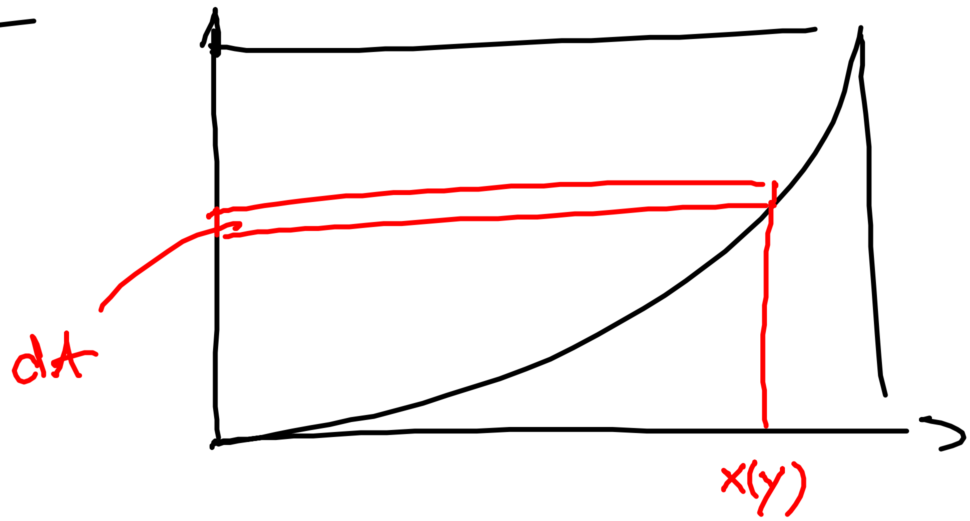
$$= \frac{\left[\frac{1}{2}ax^2 - \frac{1}{4a}x^4\right]_0^a}{\left[ax - \frac{1}{3a}x^3\right]_0^a}$$

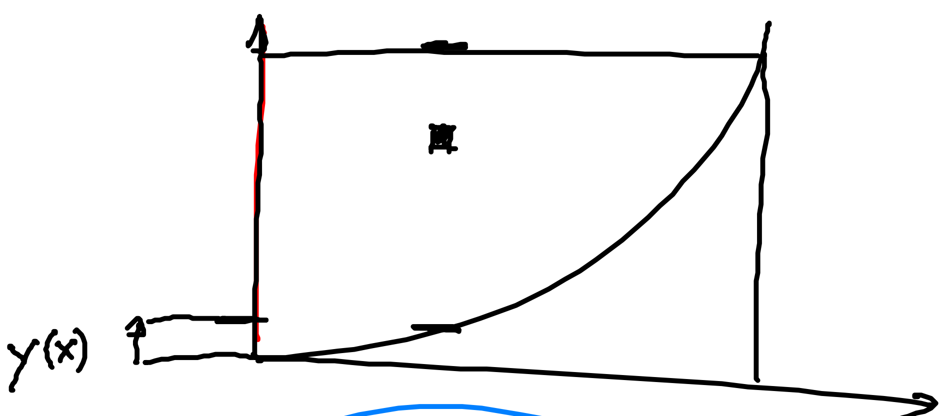
~~[1] [m] 1+l~~

$$= \frac{\left[\frac{1}{2}a^3 - \frac{1}{4}a^3 - 0 + 0\right]}{\frac{3}{3}a^2 - \frac{1}{3}a^2}$$

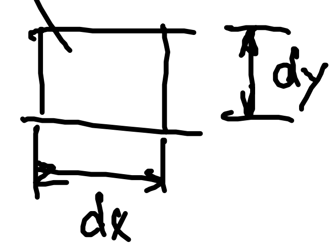
$$= \frac{\frac{1}{4}a^{\cancel{3}}}{\frac{2}{3}\cancel{a^2}} = \frac{1}{4}a \cdot \frac{3}{2} = \frac{3}{8}a$$

$y_s =$





$dA = dx\,dy$



$$y_s = \frac{\iint y\,dx\,dy}{\iint dx\,dy}$$

I_1

$$\int dA = \frac{2}{3}a^2$$

$$I_1 = \int_0^a \left[\int_{y(x)}^a y \; dy \right] dx$$

$$= \int_0^a \left[\frac{1}{2} y^2 \right]_{y(x)}^a dx$$

$$= \int_0^a \left[\frac{1}{2} a^2 - \frac{1}{2} \underbrace{y(x)^2}_{= \frac{1}{a} x^2} \right] dx$$

$$= \int_0^a \left[\frac{1}{2} a^2 - \frac{1}{2a^2} x^4 \right] dx$$

$$= \left[\frac{1}{2} a^2 x - \frac{1}{10 a^2} x^5 \right]_0^a$$

$$= \frac{1}{2} a^3 - \frac{1}{10} a^3 - 0 + 0$$

$$I_1 = \frac{2}{5} a^3$$

$$y_s = \frac{\frac{2}{5} a^3}{\int dA} = \frac{\cancel{2}}{5} a^3 \cdot \frac{3}{\cancel{2} a^2} = \underline{\underline{\frac{3}{5} a}}$$