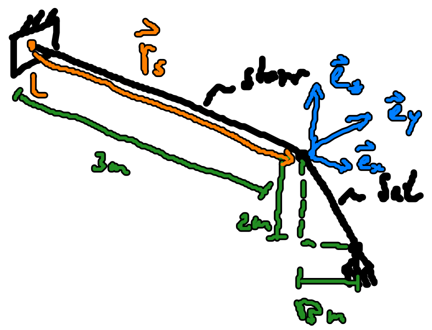
Kraftvektoren

$$\vec F = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \underbrace{F_x}_{\text{Komponenten}} \overbrace{\vec e_x}^{\text{Basisvektor}} + F_y \vec e_y + F_z \vec e_z$$



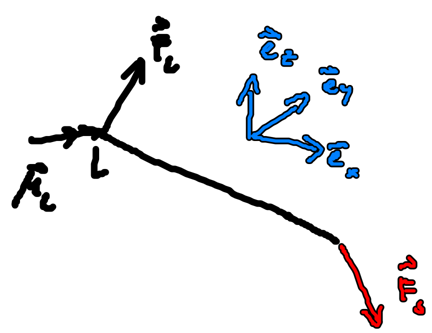
geg: Kraft im Seil sei $F_S = 30$ N

ges: Lagerreaktionen

1) skalare Lösung: ⇝ Vorstellungskraft

2) vektorielle Lösung: ⇝ Mathe

1) FS:



2) GGB:

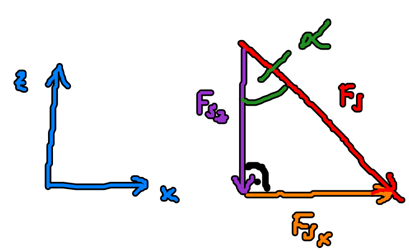
$$\sum \vec F_i = 0 \Rightarrow \vec F_L = -\vec F_S$$

$$\sum \vec M_i^L = 0 \Rightarrow \vec M_L = -\vec M_S^L$$

3.) $\vec F_S = ?$

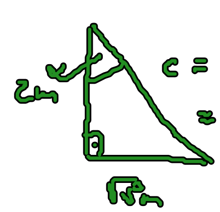
i) $\vec F_S = \begin{pmatrix} F_{Sx} \\ F_{Sy} \\ F_{Sz} \end{pmatrix}$

Skizze



$F_{Sx} = F_S \sin(\alpha)$

$F_{Sz} = -F_S \cos(\alpha)$



$c = \sqrt{2^2 + 5}$ m $= 3$ m

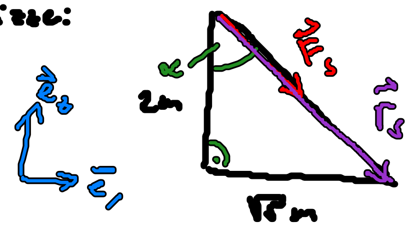
$\sin(\alpha) = \frac{\sqrt5}{3}$

$\cos(\alpha) = \frac{2}{3}$

$$\Longrightarrow \quad \vec{F}_S = F_S \begin{pmatrix} \frac{\sqrt{5}}{3} \\ 0 \\ -\frac{2}{3} \end{pmatrix}$$

ii) $\vec{F}_S = \underbrace{F_S}_{\text{Betrag}} \cdot \overbrace{\vec{e}_S}^{\text{Richtung}\ |\vec{e}_S| = 1}$

Skizze:



$$\vec{r}_S = \begin{pmatrix} \sqrt{5}\,m \\ 0 \\ -2m \end{pmatrix}$$

$$\vec{e}_S = \frac{\vec{F}_S}{\|\vec{F}_S\|}$$

$$\vec{e}_S = \frac{1}{3\cancel{m}} \begin{pmatrix} \sqrt{5}\,\cancel{m} \\ 0 \\ -2\cancel{m} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{5}}{3} \\ 0 \\ -\frac{2}{3} \end{pmatrix} \Longrightarrow \vec{F}_S = F_S \cdot \begin{pmatrix} \frac{\sqrt{5}}{3} \\ 0 \\ -\frac{2}{3} \end{pmatrix}$$

4.) LR:

$$\vec{F}_L = -\vec{F}_S = F_S \begin{pmatrix} -\frac{\sqrt{5}}{3} \\ 0 \\ \frac{2}{3} \end{pmatrix}$$

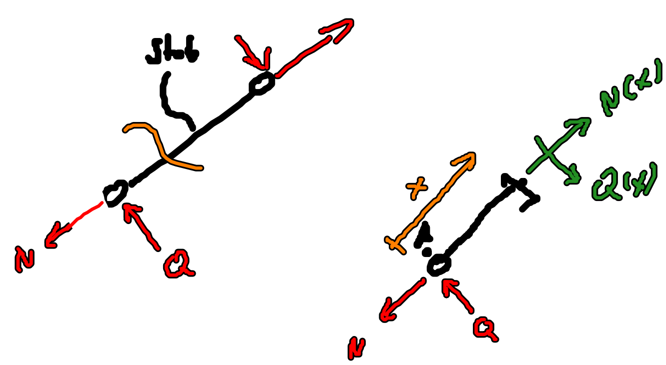
$$\vec{M}_L = -\vec{M}_S^L = -(\vec{r}_S \times \vec{F}_S) = -\left(3m\, \overbrace{\underbrace{\vec{e}_x \times F_S \left(\frac{\sqrt{5}}{3} \vec{e}_x\right.}_{=0} + \left.\left(-\frac{2}{3}\right) \vec{e}_z \right)}^{= -\vec{e}_y}\right)$$

$$\vec{M}_L = -\cancel{3}m \frac{2}{\cancel{3}} F_S \vec{e}_y = -2 \cdot F_S\, m\, \vec{e}_y = -60\ \mathrm{Nm}\ \vec{e}_y$$

Fachwerke:

ideales FW:

- gelenkig gelagert
- Kräfte nur in Knoten



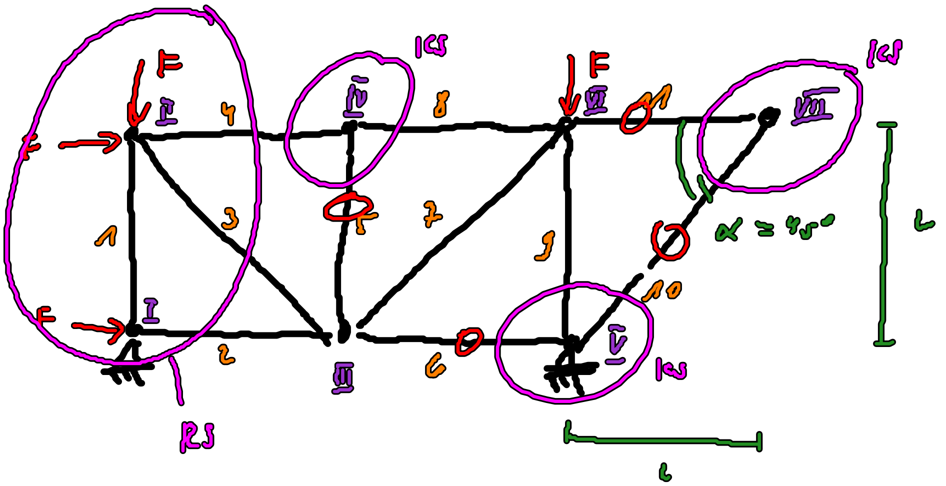
Stäbe übertragen nur $\Leftarrow$

x: $N(x) = N = \text{konst.}$

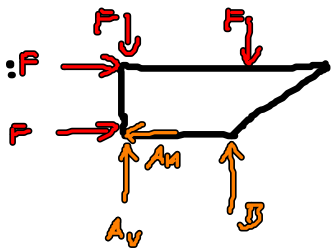
z: $Q(x) = Q = 0$

Normalkräfte

$\curvearrowright A: \; Q(x) \cdot x = 0 \implies Q(x) = 0$



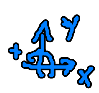
LR:

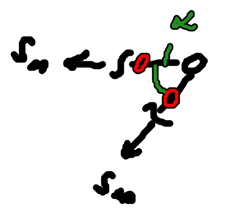


GGB: $B = \frac{3}{2} F, \; A_V = \frac{1}{2} F, \; A_H = 2F$

1) Nullstäbe

KS: VII



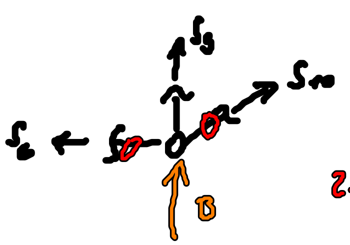


$x: \; 0 = -S_{11} - S_{10} \cos(\alpha) \implies S_{11} = -S_{10} \cos(\alpha) = 0$

$y: \; 0 = -S_{10} \sin(\alpha) \implies S_{10} = 0$

1. NST-Regel

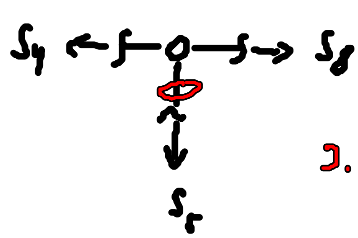
KS: V



$x: \; 0 = -S_6 \implies S_6 = 0$

2. NST-Regel

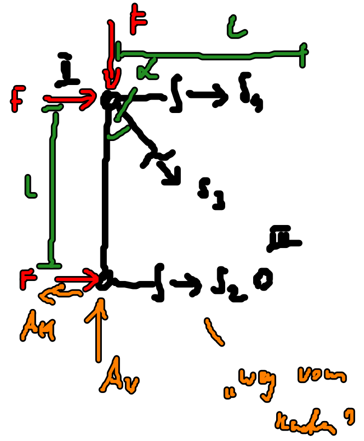
KS: IV



$y: \; 0 = -S_5 \implies S_5 = 0$

3. NST-Regel

2) RITTER-Schnitt



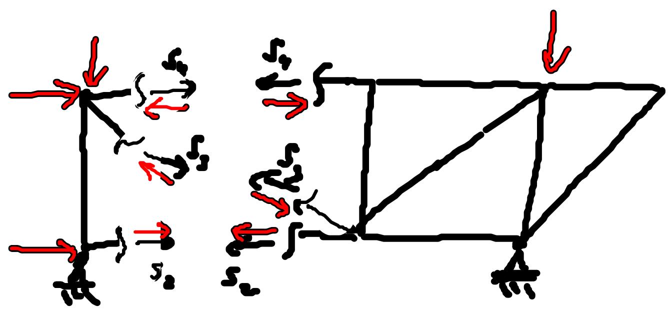
$$M^{III}: \quad 0 = -A_V \cancel{l} - \cancel{Fl} - S_4 \cancel{l} + \cancel{Fl}$$

$$S_4 = -A_V = -\frac{1}{2}F \quad (D)$$

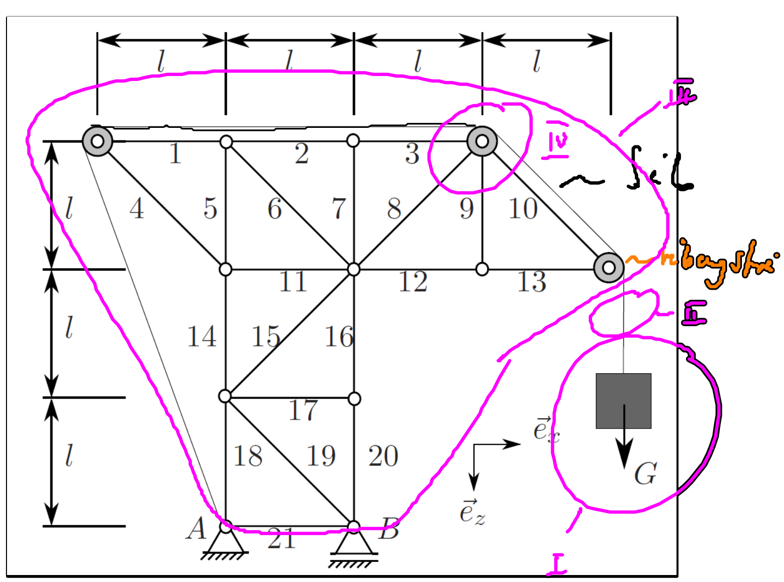
$$y: \quad 0 = A_V - F - S_3 \frac{\sqrt{2}}{2}$$

$$S_3 = \frac{2}{\sqrt{2}}(A_V - F) = -\frac{\cancel{2}}{\sqrt{2}} \frac{1}{\cancel{2}} F = -\frac{1}{\sqrt{2}}F \quad (D)$$

$$M^{I}: \quad 0 = S_2 \cancel{l} + F\cancel{l} - A_H \cancel{l} \Rightarrow S_2 = A_H - F = +F \quad (Z)$$



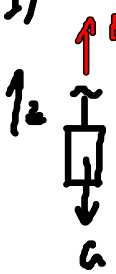
Seile im System



Seil:

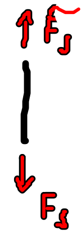
- Nur Kräfte entlang seiner Richtung
- Nur Zugkräfte
- biegeweich

I)



z: $F_S = G$

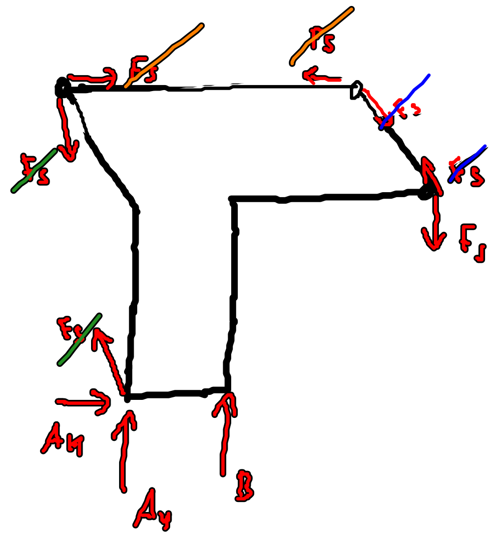
II)



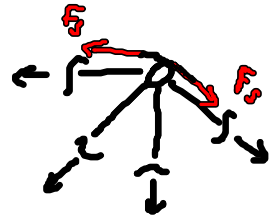
z: $\hat{F}_S = F_S$ = const.

Seilkraft im ganzen Seil: F_S

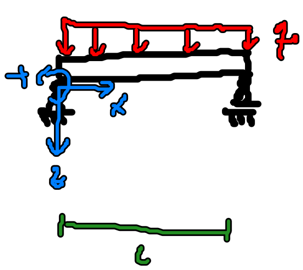
III)



IV)



Schnittlasten



ges: Q, M

=> SL-Differentialgleichungen

$$\frac{dQ}{dx} = Q' = -q \qquad (1)$$

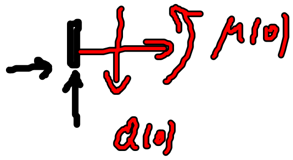
$$\frac{dM}{dx} = M' = Q \qquad (2)$$

(1): $Q(x) = -\int q(x)\,dx + C_1 = -q_0 x + \underline{C_1}$

(2): $M(x) = \int Q(x)\,dx + C_2 = -\frac{1}{2} q_0 x^2 + C_1 x + \underline{C_2}$

C_1 / C_2 aus RBen: („Kenne ich M(x) oder Q(x) am Rand?")

x = 0:



$M(0) = 0$ RB1

x = L

$M(L) = 0$ RB2

RBen einsetzen:

RB 1: $0 = -\cancel{\frac{1}{2} q_0}\, 0^2 + \cancel{C_1} \cdot 0 + C_2 \Rightarrow C_2 = 0$

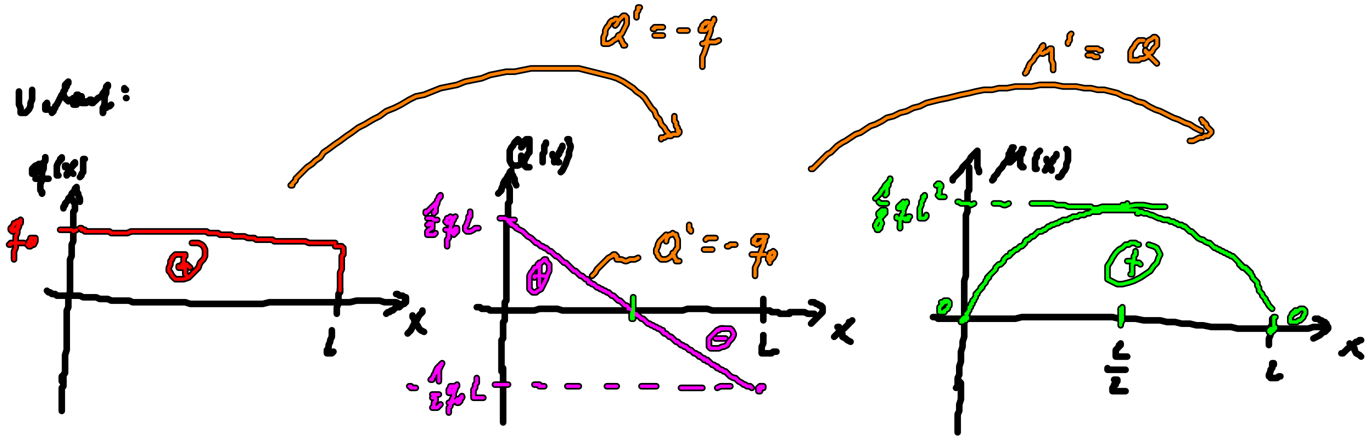
RB 2: $0 = -\frac{1}{2} q_0 L^2 + C_1 L \Rightarrow C_1 = \frac{1}{2} q_0 L$

Ergebnis:

$$Q(x) = -q_0 x + \frac{1}{2} q_0 L = q_0 L\left(\frac{1}{2}\cancel{L} - \frac{x}{L}\right) = q_0 L\left(\frac{1}{2} - \left(\frac{x}{L}\right)\right)$$

$$M(x) = -\frac{1}{2} q_0 x^2 + \frac{1}{2} q_0 L x = \frac{1}{2} q_0 L^2\left(\frac{\cancel{L}x}{L^2} - \frac{x^2}{L^2}\right) = \frac{1}{2} q_0 L^2\left(\left(\frac{x}{L}\right) - \left(\frac{x}{L}\right)^2\right)$$

Verlauf:



$Q(0) = \frac{1}{2} q_0 L$

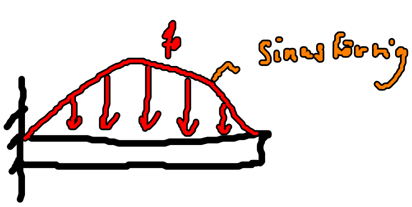
$Q(L) = -\frac{1}{2} q_0 L$

$M(0) = 0$

$M(L) = 0$

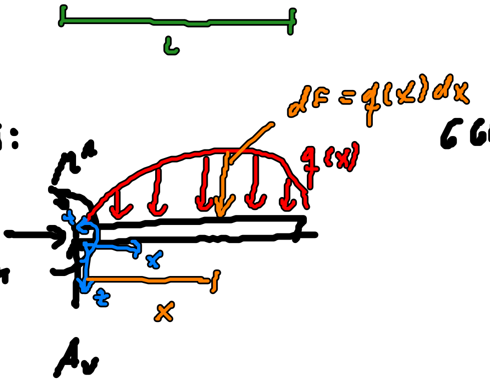
$M\left(\frac{L}{2}\right) = \frac{1}{8} q_0 L^2$

Sammlung



$$q(x) = q_0 \sin\left(\frac{\pi}{L} x\right)$$

FB:



GGB:

$$z: \quad 0 = -A_V + \int_F dF$$

$$A_V = \int_0^L q(x)\,dx = \int_0^L q_0 \sin\left(\frac{\pi}{L}x\right)dx$$

$$A_V = \left[-\frac{L}{\pi} q_0 \cos\left(\frac{\pi}{L}x\right)\right]_0^L$$

$$A_V = -q_0 \frac{L}{\pi}\left(\underbrace{\cos(\pi)}_{=-1} - \underbrace{\cos(0)}_{=1}\right) = 2 q_0 \frac{L}{\pi}$$

$$M^A: \quad 0 = M^A - \int_M dM \implies M^A = \int_0^L x\, q(x)\,dx$$

$$M^A = -x q_0 \frac{L}{\pi} \cos\left(\frac{\pi}{L}x\right)\Big|_0^L + \int_0^L q_0 \frac{L}{\pi} \cos\left(\frac{\pi}{L}x\right)dx = q_0 \frac{L^2}{\pi}$$

$$\Longrightarrow \quad x_s = \frac{\int_0^L x\, q(x)\,dx}{\int_0^L q(x)\,dx} = \frac{\cancel{q_0} \frac{L^{\cancel{2}}}{\cancel{\pi}}}{2\cancel{q_0} \frac{\cancel{L}}{\cancel{\pi}}} = \frac{1}{2} L$$

Angriffspunkt der Resultierenden